



Work Sheet N°01: Number Systems

Exercise 01:

1) Write each of the following numbers in its polynomial form.

Example:

$$125_{10} = 1 \times 10^2 + 2 \times 10^1 + 5 \times 10^0$$

$$3409_{10} = ? , \quad 1011_2 = ? , \quad 461_7 = ? , \quad 237_8 = ? , \quad 2A0C_{16} = ? .$$

2) Deduce the polynomial form in the general case for a number $N = (a_n a_{n-1} \dots a_1 a_0)_b$ in radix b .

Exercise 02:

1) Count from 5_{10} to 20_{10} , in binary, octal and hexadecimal.

2) Consider the following numbers:

$$\text{a) } 101101 \quad \text{b) } 728 \quad \text{c) } 3A5 \quad \text{d) } 764 \quad \text{e) } 10021$$

a) Which radices (bases) are possible for each number?

b) Which numbers cannot be decimal, octal, or binary?

Exercise 03:

Consider the number: 1011. Calculate its decimal value when it is interpreted as:

a) A binary number 1011_2 .

b) An octal number 1011_8 .

c) A hexadecimal number 1011_{16} .

Exercise 04:

Find the radix b in each case:

$$\text{a) } (12)_b = 17_8$$

$$\text{b) } (23)_b = 11_{12}$$

$$\text{c) } (123)_b = 38_{10}$$

$$\text{d) } (1001)_b = 201_{16}$$

Exercise 05:

1) Encode the following decimal numbers to binary, octal and hexadecimal:

$$56, \quad 100, \quad 127, \quad 255, \quad 256.$$

2) Decode the following numbers:

$$(101011)_2, \quad (10000000)_2, \quad (103)_4, \quad (1051)_6, \quad (9E7)_{16}, \quad (65)_8$$

3) Perform the following conversions (transcoding):

$$\text{i) } (1010111010)_2 = (\quad)_8 = (\quad)_{16}$$

ii) $(F04)_{16} = (\quad)_2 = (\quad)_8$

iii) $(237)_8 = (\quad)_2 = (\quad)_{16}$

Exercise 06:

1) Encode the following fractional numbers into binary, octal and hexadecimal :

$$21, 25 \quad ; \quad 64, 16 \quad ; \quad 19, 1 \quad ,$$

using the successive divisions and multiplications for each conversion. Indicate whether the fractional part has a finite, infinite, or periodic sequence of digits.

2) Give the decimal value of:

$$(1001, 011)_2 \quad ; \quad (10, 014)_8 \quad ; \quad (A0, 2)_{16}$$

3) Give the octal and the hexadecimal representations of:

$$(10101, 0011011)_2 \quad ; \quad (11101001, 001)_2$$

4) Give the binary value of:

$$(2C1F, E1)_{16} \quad ; \quad (160, 4)_8$$

Exercise 07:

1) Perform the following arithmetic operations in binary:

- | | |
|----------------------------|----------------------------|
| a) $(11001)_2 + (1111)_2$ | b) $(16)_{16} + (20)_8$ |
| c) $(30)_{10} - (15)_{10}$ | d) $(101101)_2 - (1011)_2$ |
| e) $(25)_8 * (31)_4$ | f) $(111010)_2 / (5)_{10}$ |

2) Is it possible to perform the following binary addition on 1 byte?

$$(145)_{10} + (111)_{10}$$

Exercise 08–Optional:

Let b be an unknown radix. Consider: $A = (132)_b$, $B = (24)_b$, and suppose that

$$A = (56)_{10}.$$

1. Determine the radix b .
2. Convert A and B into binary.
3. Calculate: $A + B$ directly in binary.
4. Calculate: $A - B$ directly in binary.
5. Convert both results into hexadecimal.
6. Verify the two operations using decimal representations.

Exercise 09–Optional:

Consider a number N in a radix- x system, defined by

$$N = 2 \cdot x^3 + 3 \cdot x^2 + 5 \cdot x + 2.$$

1. Give the numerical representation of N in radix- x .
2. Show that N is a multiple of 21_x .
3. Assume that $x = 8$.
 - (a) Convert N to binary.
 - (b) Convert N to hexadecimal.