**Final Examination – Semester 3** | **Master 2 – All Tracks** | **2025–2026**

**Module:** English for Technical Purposes **Duration:** 1 hour
**Instructions:** Answer clearly. Use correct mathematical English.

# Exercise 1: Mathematical Logic (6 pts)

Consider the following propositions:

$$p : \text{“I leave.”},\ q\ : \text{“You stay.”},\ r : \text{“There is no one here.”}$$

Translate the following logical formulas into French, then construct their corresponding truth table(s):

(a) $(p \wedge \neg q) \Rightarrow r$ (b) $(\neg p \vee \neg q) \Rightarrow \neg r$

# Exercise 2: ODE (6.5 pts)

Solve the following ordinary differential equation:

$$x^2 y + \sqrt{1+x^3}\, y' = 0.$$

# Exercise 3: Reading Comprehension (7.5 pts)

**Text:** Excerpt from *“Mathematical Reasoning or Not?”*

Mathematical reasoning is often considered the highest form of rigorous thinking. Unlike everyday reasoning, it is based on formal systems defined by axioms and strict rules of deduction. While other forms of reasoning such as induction, analogy, and dialectics play important roles in science and education, they do not meet the standards required for mathematical proof. Mathematical reasoning aims to eliminate ambiguity, coincidence, and subjective interpretation. However, despite its rigor, it is not a perfect system, as shown by Gödel’s incompleteness theorems, which demonstrate inherent limitations in any sufficiently powerful axiomatic framework.

**Questions (1.5 pts each)**

1. What is the main objective of mathematical reasoning according to the text?
2. How does mathematical reasoning differ from everyday reasoning?
3. Why are analogy and dialectics insufficient for mathematical proof?
4. What do Gödel’s incompleteness theorems highlight?
5. Does the text present mathematical reasoning as flawless?

# Correction of Exercise 1

1- English translations:

- (a) $(p \wedge \neg q) \Rightarrow r$: "If I leave and you don't stay, then there's no one here!". Traduction en français : « Si je pars et que tu ne restes pas, alors il n'y a plus personne ici ! » (1.5 pts)
- (b) $(\neg p \vee \neg q) \Rightarrow \neg r$: "If I don't leave or you don't stay, then someone is here!". raduction en français : « Si je ne pars pas ou que tu ne restes pas, alors quelqu'un est ici ! » (1.5 pts)

2- Truth table (3 pts)

| $p$ | $q$ | $r$ | $\neg p$ | $\neg q$ | $p \wedge \neg q$ | $(p \wedge \neg q) \Rightarrow r$ | $\neg p \vee \neg q$ | $\neg r$ | $(\neg p \vee \neg q) \Rightarrow \neg r$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |

# Correction of Exercise 2

We rewrite the equation in differential form:

$$\sqrt{1+x^3}\,\frac{dy}{dx} = -x^2 y.$$

(1 pt)

Separating the variables gives

$$\frac{1}{y}\,dy = -\frac{x^2}{\sqrt{1+x^3}}\,dx.$$

(2 pts)

Integrating both sides,

$$\int \frac{1}{y}\,dy = -\int \frac{x^2}{\sqrt{1+x^3}}\,dx.$$

For the right-hand side, we use the substitution

$$u = 1 + x^3, \quad du = 3x^2\,dx.$$

Thus,

$$-\int \frac{x^2}{\sqrt{1+x^3}}\,dx = -\frac{1}{3}\int \frac{du}{\sqrt{u}} = -\frac{2}{3}\sqrt{1+x^3}.$$

(1 pt)

Hence,

$$\ln|y| = -\frac{2}{3}\sqrt{1+x^3} + K, K \in \mathbb{R}.$$

(2 pts)

Exponentiating, the general solution is

$$\boxed{y(x) = C\,\exp\left(-\frac{2}{3}\sqrt{1+x^3}\right),}$$

where $C$ is a nonzero constant. (0.5 pts)

# Answers to Exercise 3

This is a suggested set of answers; any correct answer will be accepted.

1. The main objective of mathematical reasoning is to achieve rigorous, unambiguous conclusions through formal deduction. [Or: It aims to eliminate ambiguity, coincidence, and subjective interpretation] (1.5 pts)
2. Mathematical reasoning relies on axioms and strict logical rules, whereas everyday reasoning is informal and often influenced by intuition or experience. [Or: It is based on formal systems defined by axioms and strict rules of deduction.] (1.5 pts)
3. Analogy and dialectics lack deductive certainty and therefore cannot guarantee universally valid conclusions required for mathematical proofs. [Or: They do not meet the standards required for mathematical proof]. (1.5 pts)
4. Gödel's incompleteness theorems show that Mathematical reasoning is not a perfect system. (1.5 pts)
5. No, the text explicitly states that mathematical reasoning has inherent limitations despite its rigor. (1.5 pts)