Algebra Guided Exercises -- Academic Year 2025/2026

Elements of logic – Part2 Series 1

Exercise1 : Express the following propositions using quantifiers :

1- The function f is zero for all real x.
2- The function f is zero at x_0
3- f always takes the value 2
4- f takes the value 2 at least once

Exercise2 : Say if truth or false and give the negation :

1- $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}; 2x + y > 3$
2- $\forall \epsilon > 0, \exists \alpha > 0; |x| < \alpha \Rightarrow |x^2| < \epsilon$
3- $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}; y^2 > x$
4- $\exists\, n \in \mathbb{N}, \forall x \in \mathbb{R}; x + 1 \geq n$
5- $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}; (2x + y > 0 \; and \; 2x + y = 0)$

Exercise3 : Using contradiction prove that :

1- $\sqrt{2} \notin \mathbb{Q}$
2- $\forall n \in \mathbb{N}; n^2 \; even \Rightarrow n \; even$

Exercise4 : Using the contrapostive form show that :

1- $(n^2 - 1) \; is \; not \; divisible \; by \; 8 \Rightarrow n \; is \; even$
2- $(\forall \epsilon > 0 \; |x| \leq \epsilon) \Rightarrow x = 0$