University of Tlemcen **Academic year 2024-2025**
Faculty of Sciences **(L2 Maths)**
department of mathematics **Algebra 3 (Second Year)**

Worksheet N°4/ " Reduction of endomorphisms "

Exercise 01: Let E be a $K-$vector space, with finite dimension n and f an endomorphism defined in E. Show that:

$$0 \notin Sp\ (f\ ) \Leftrightarrow f \text{ is surjective.}$$

Exercise 02: Let E be a $K-$vector space, with finite dimension n.
Show that if 0 is an eigenvalue of f^n, such that $f^n = \underbrace{f \circ ... \circ f}_{n \text{ times}}$, then 0 is an eigenvalue of f.

Exercise 03: Let f be an endomorphism defined in $\mathbb{C}[X]$ by $f(P(X)) = (X-1)P(X)$. Show that f has no eigenvalues.

Exercise 04: Let f and g be two endomorphisms defined in E a $K-$vector space, with finite dimension n. Show that $f \circ g$ and $g \circ f$ have the same eigenvalues.

Exercise 05: Let E a vector space of functions C^∞ on $\mathbb{R}$ with values in $\mathbb{R}$. Φ is an endomorphism defined in E by:

$$\begin{array}{rcl} \Phi & : & E \to E \\ f & \mapsto & \Phi(f)(x) = \displaystyle\int_0^1 e^{x-t} f(t)dt. \end{array}$$

Find the eigenvalues and the eigenvectors of Φ.

Exercise 06: Let E a vector space of functions C^∞ on $\mathbb{R}$ with values in $\mathbb{R}$. Φ is an endomorphism defined in E by:

$$\begin{array}{rcl} \Phi & : & E \to E \\ f & \mapsto & \Phi(f)(x) = \displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x+t) f(t)dt. \end{array}$$

Find the eigenvalues and the eigenvectors of Φ.

Exercise 07: Let be $\mathbb{R}_2[X]$ the $\mathbb{R}-$space vector of polynomials of degree ≤ 2 and f an endomorphism defined by:

$$\begin{array}{rcl} f & : & \mathbb{R}_2[X] \to \mathbb{R}_2[X] \\ P & \mapsto & f(P) = 3 + (2X+1)P' + (X^2-4)P''. \end{array}$$

Calculate $f^n(a_0 + a_1 X + a_2 X^2)$.

Exercise 08:

Sincere wishes you success (MESSIRDI BACHIR)