**University of Tlemcen** **Academic year 2024-2025**
**Faculty of Sciences** **(L2 Maths)**
department of mathematics **Algebra 3 (Second Year)**

**Worksheet N°3/ " Systems of linear equations "**

Exercise 01: Solve the following systems:

$$(S_1)\begin{cases} 4x+2y-2z=4 \\ 7x+3y+z=-3 \\ 6x+2y-3z=5. \end{cases} \quad (S_2)\begin{cases} x+2y-3z=1 \\ 2x+3y-4z=-2 \\ 5x+y-9z=4 \\ 6x+2y-z=-3. \end{cases} \quad (S_3)\begin{cases} -2x+5y-z-4t=3 \\ 3x+7y+2z+t=-2 \\ 4x+y-5z-2t=4. \end{cases}$$

Exercise 02: (1) Calculate the sum $S=x+y+z$ ($(x,y,z)$ is the solution) without solving the following systems:

$$(S)\begin{cases} 4x+7y+5z=5 \\ 2x-y+3z=-1 \\ x+4y-z=-5. \end{cases}$$

(2) Find the solutions of the following equation:

$$3x+5y-2z=0.$$

Exercise 03: (1) Discuss the following system solutions according to the real parameter $m$ :

$$(S_1)\begin{cases} 2x+y+(3+m)z=3+m \\ x-(1-m)y+2z=-1 \\ 3x+4y-3z=1-m. \end{cases}$$

(2) Solve the following system:

$$(S_2)\begin{cases} x+ay-a^2z=a^4 \\ x+by-b^2z=b^4 \\ x+cy-c^2z=c^4 \end{cases} \quad a,b,c\in\mathbb{R}.$$

(3) Discuss the following system solutions according to the real parameter $a$ :

$$(S_3)\begin{cases} x+y+z+at=a \\ x+y+az+t=1 \\ x+ay+z+t=a^2. \end{cases}$$

Exercise 04: (1) In $\mathbb{R}_2[X]$ the $\mathbb{R}$ space vector (set of polynomials of degree $\leq 2$), let $f$ be an endomrphism defined by:

$$f(P)=2P(X)-(2X+1)P'(X)+Xp''(X)+P'(0)(3-2X^2).$$

Find the set of polynomials that verifies:

$$f(P)=X^2-2X+4.$$

(2) In $\mathbb{R}_3[X]$ the $\mathbb{R}$ space vector (set of polynomials of degree $\leq 3$), find the tree real numbers $a, b, c$ such that:

$$\int_2^5 P(x)dx = aP(1) + bP(2) + cP(4).$$

Exercise 05: Let $F$ be a subspace of $\mathbb{R}^4$ defined by:

$$F = \left\{(x, y, z, t) \in \mathbb{R}^4 : 2x - 3y + z + 7t = 0\right\}.$$

And let $V \in \mathbb{R}^4$, defined by:

$$V = \begin{pmatrix} -1 \\ 2 \\ 1 \\ 1 \end{pmatrix}.$$

Verify that $V$ belongs to $F$ and find the components of $V$ in the basis of the subspace $F$.

*Sincere wishes you success (MESSIRDI BACHIR)*