



## Tutorial Series 1 - Math2

### Exercise 1

Perform the following products when possible. If not, explain why.

$$\text{a) } \begin{pmatrix} 2 & 5 \\ 3 & 6 \\ 4 & 7 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 4 & 6 \end{pmatrix}, \text{ b) } \begin{pmatrix} 2 & 5 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 3 & 6 \\ 4 & 7 \end{pmatrix} \text{ c) } (-1 \ 4 \ 5) \begin{pmatrix} 0 & -1 & 6 \\ 2 & 4 & -2 \\ 3 & 5 & 3 \end{pmatrix}, \text{ d) } \begin{pmatrix} 1 & -1 \\ 2 & 0 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 3 & 6 \\ 4 & 1 \end{pmatrix}.$$

### Exercise 2

Let,  $A, B, C$  the following matrices:

$$A = \begin{pmatrix} 0 & 1 & -1 \\ -3 & 4 & -3 \\ -1 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 0 & 5 \\ -6 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & -3 \\ 2 & -1 \\ 8 & -7 \end{pmatrix}.$$

Calculate  $2B + 3C, A^2, A^3, {}^t(AB)$ .

### Exercise 3

Calculate the determinants of the following matrices:

$$1) \begin{pmatrix} 7 & 11 \\ -8 & 4 \end{pmatrix} \quad 2) \begin{pmatrix} 1 & 0 & 6 \\ 3 & 4 & 15 \\ 5 & 6 & 21 \end{pmatrix}, \quad 3) \begin{pmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{pmatrix}, \quad 4) \begin{pmatrix} 1 & 0 & -1 \\ 2 & 3 & 5 \\ 4 & 1 & 3 \end{pmatrix}, \quad 5) \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{pmatrix}.$$

### Exercise 4

$$\text{Let } M = \begin{pmatrix} 0 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

- 1) Calculate  $M^3 - 2M^2 + 2M$ .
- 2) Deduce from the above that the matrix  $M$  is invertible, then provide  $M^{-1}$ .

### Exercise 5

- 1) Show that the matrix  $P = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$ , is invertible and calculate  $P^{-1}$ .

$$2) \text{ Let a matrix } A = \begin{pmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{3}{5} \end{pmatrix}$$

. Find a diagonal matrix  $D$  such that  $A = PDP^{-1}$ .