Exercise Serie No. 2

Exercise 1:

In each of the following questions, we are given a set E and subsets A and B of E. Explicitly determine the sets $A \cap B, A \cup B, A \cap C_E(B)$ and $C_E(A) \cap B$.

1. $E = \{1, 2, 3, 4\}, \quad A = \{1, 2\}, \quad B = \{2, 4\}$.
2. $E = \mathbb{R}, \quad A =]-\infty, 2], \quad B = [3, +\infty[$.

Exercise 2:

Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$.
Determine the following sets:
$f([0, 1[), \quad f(]-1, 2[), \quad f^{-1}([1, 2[), \quad f^{-1}([-1, 1]), \quad f^{-1}(\{3\})$.

Exercise 3:

Are the following functions injective, surjective, bijective?

1. f from $\mathbb{R}$ to $[0, +\infty[$ defined by $f(x) = x^2$.
2. g from $[0, +\infty[$ to $[0, +\infty[$ defined by $g(x) = x^2$.

Exercise 4:

Let h be the function from $\mathbb{R}$ to $\mathbb{R}$ defined by $h(x) = \dfrac{4x}{x^2 + 1}$.

1. Verify that for any nonzero real number a, we have $h(a) = h(\frac{1}{a})$. Is the function h injective?
2. Let f be defined on $I = [1, +\infty[$ by $f(x) = h(x)$.
   (a) Show that f is injective.
   (b) Verify that: $\forall x \in I, f(x) \leq 2$.
3. Show that f is a bijection from I onto $]0, 2]$ and find f^{-1}.