Exercise Series N°1

Exercise 1: *Direct Reasoning*

- Is the product of two even numbers necessarily even?
- Is the product of two odd numbers necessarily odd? **(SUPP)**

Exercise 2: *Contrapositive Reasoning*

Prove using the contrapositive method that:

- $(\forall \varepsilon > 0,\ |x| \leq \varepsilon) \implies x = 0$.
- If $(n^2 - 1)$ is not divisible by 8, then n is even.

Exercise 3: *Proof by Contradiction*

Prove by contradiction that:

- $\forall n \in \mathbb{N}$, n^2 even $\implies$ n is even.
- $\sqrt{2} \notin \mathbb{Q}$. **(SUPP)**

Exercise 4: *Counterexample Reasoning*

Show that the following assertions are false:

- $\forall x \in \mathbb{R}$, $x^2 \geq 1$.
- Every even integer is the product of two even numbers.

Exercise 5: *Proof by Mathematical Induction*

Prove by induction that:

1. $\forall n \in \mathbb{N}^* : 1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$.
2. $\forall n \in \mathbb{N}^*,\ 4^n + 6n - 1$ is divisible by 9. **(SUPP)**