Exercise 1.
Check the Reflexivity, Symmetry and Transitivity properties of the following binary relations :

1. $\forall x, y \in \mathbb{Z},\ x\mathcal{R}y \Longleftrightarrow (x-y)$ *is a multiple of* 2 ***and*** *of* 3.
2. $\forall x, y \in \mathbb{Z},\ x\mathcal{P}y \Longleftrightarrow (x-y)$ *is a multiple of* 2 ***or*** *of* 3.
3. $\forall (a, a^{'})(b, b^{'}) \in \mathbb{N} \times \mathbb{N},\ (a, a^{'})\mathcal{S}(b, b^{'}) \Longleftrightarrow a + a^{'} = b + b^{'}$.

Among the previous relations, are there equivalence relations ?

Exercise 2.
Show that the relation $\mathcal{P}$ defined on $\mathbb{R}$ by :

$$x\mathcal{P}y \Longleftrightarrow \cos^2 x + \sin^2 y = 1$$

is an equivalence relation.

Exercise 3.
Let $\mathcal{R}$ be the relation defined on $\mathbb{Z}$ by :

$$x\mathcal{R}y \Longleftrightarrow \exists m \in \mathbb{Z},\ x - y = 7m$$

1. *Show that $\mathcal{R}$ is an equivalence relation on $\mathbb{Z}$.*
2. *What are the equivalence classes ?*
3. *Determine the quotient set.*

Exercise 4.
Let $\mathcal{T}$ be the relation defined on $\mathbb{N}^*$ by :

$$p\mathcal{T}q \Longleftrightarrow \exists n \in \mathbb{N}^*,\ p^n = q$$

1. *Show that $\mathcal{T}$ is an order relation.*
2. *Is this order total or partial ?*