University of Tlemcen **Academic Year 2024-2025**
Faculty of Sciences
Computer Science Department

L1 Ing Info ALGEBRA 1

Tutorial sheet 3 : Binary relations

Exercise 1 :
Let $\mathcal{R}$ be the relation defined on $\mathbb{R}^*$ by :

$$x\mathcal{R}y \Leftrightarrow x^2 - \frac{1}{x^2} = y^2 - \frac{1}{y^2}.$$

1. Prove that $\mathcal{R}$ is an equivalence relation.
2. Determine the equivalence class of $a \in \mathbb{R}^*$.

Exercise 2 :
Let $\mathcal{R}$ be the relation defined on $\mathbb{Z}$ by :

$$a\mathcal{R}b \Leftrightarrow 3 \text{ divise } a - b.$$

1. Prove that $\mathcal{R}$ is an equivalence relation.
2. Determine the equivalence class of $a \in \mathbb{Z}$. Calculate $\dot{0}, \dot{1}, \dot{2}$ and $\dot{3}$.
3. Deduce the quotient set $\mathbb{Z}/\mathcal{R}$ denoted by $\mathbb{Z}/3\mathbb{Z}$.

Exercise 3 :
Let the following relation defined on $\mathbb{R}^2$ by :

$$(x, y)\mathcal{R}(x', y') \Leftrightarrow (x \leqslant x' \text{ and } y \leqslant y').$$

1. Prove that $\mathcal{R}$ is an order relation. This is total order ?
2. Specify two upper bounds, two lower bounds, the supremum, and the infimum of the set $A = \{(1, 2), (3, 1)\}$.
3. Does the set A have a greatest and smaller element ?

Exercise 4 :
Let Φ be a relation defined on $\mathbb{N}^*$ by :

$$x\Phi y \Leftrightarrow \exists n \in \mathbb{N}^* \text{ such that } x^n = y.$$

1. Prove that Φ is an order relation on $\mathbb{N}^*$.
2. This is total order ?
3. Let $A = \{1, 4, 8\}$. Determine whether Max$A$ and Min$A$ exist for the order Φ.

https ://dmath.univ-tlemcen.dz/fr/pages/190/l1-info-1er-ing-24-25