**University of Tlemcen**
**Faculty of Sciences**
**Computer Science Department**

**Academic Year 2024-2025**

L1 Ing Info — ALGEBRA 1

## Tutorial sheet 2 : Sets and applications

**Exercise 1 :**

Determine $A \cap B, A \cup B, A \cap C_E(B)$, $C_E(A) \cap B$ :

1. $E = \{1, 2, 3, 4\}, A = \{1, 2\}, B = \{2, 4\}$.
2. $E = \mathbb{R}, A = ]-\infty; 2], B = [3; +\infty[$.
3. $E = \mathbb{R}, A = \mathbb{N}, B = ]0; +\infty[$.

**Exercise 2 :**

Let $A$ be a set, and $X, Y, Z$ are subset of $A$. Prove the following properties :

a. $C_A(C_A(X)) = X$.
b. $C_A(X \cup Y) = C_A(X) \cap C_A(Y)$ and $C_A(X \cap Y) = C_A(X) \cup C_A(Y)$.
c. $X \subset Y \Leftrightarrow C_A(Y) \subset C_A(X)$.

**Exercise 3 :**

Let $f$ be an application from $E$ to $F$. Let $A, A' \subset E$ and $B, B' \subset F$. Prove that :

$$1) A \subset f^{-1}(f(A)) \qquad 2) f(f^{-1}(B)) \subset B$$
$$3) f(A \cup A') = f(A) \cup f(A') \qquad 4)\ f(A \cap A') \subset f(A) \cap f(A')$$
$$5) f^{-1}(B \cup B') = f^{-1}(B) \cup f^{-1}(B') \qquad 6) f^{-1}(B \cap B') = f^{-1}(B) \cap f^{-1}(B')$$

Show that if $f$ is injective then the equality 4) holds.

**Exercise 4 :**

Are the following applications injective, surjective or bijective ?

1. $f$ from $\mathbb{N}$ to $\mathbb{N}$ defined by $f(x) = 2x$.
2. $g$ from $\mathbb{N}$ to $\mathbb{N}$ defined by $g(x) = 2x + 1$.
3. $h$ from $\mathbb{Z}$ to $\mathbb{N}$ defined by $h(x) = |x| - [x]$ .
4. $u$ from $\mathbb{R}^+$ to $\mathbb{R}^+$ defined by $u(x) = \sqrt{x}$.

**Exercise 5 :**

Let $E, F, G$ be three sets, $f : E \to F$, $g : F \to G$ are two applications.

a. Prove that if $g \circ f$ is injective, then $f$ is injective.
b. Prove that if $g \circ f$ is surjective, then $g$ is surjective.

**Exercise 6 :**

Let $h$ be an application from $\mathbb{R}$ in $\mathbb{R}$ defined by $h(x) = \dfrac{4x}{x^2 + 1}$.

1. Verify that for all real $a \neq 0$ we have $h(a) = h(\dfrac{1}{a})$. Is $h$ injective ?
2. Let $f$ defined on $I = [1, +\infty[$ by $f(x) = h(x)$.

a. Show that $f$ is injective.
b. Verify that $\forall x \in I, f(x) \leqslant 2$.
c. Prove that $f$ is bijective from $I$ in $]0, 2]$ and determine its inverse $f^{-1}$.

https ://dmath.univ-tlemcen.dz/fr/pages/190/l1-info-1er-ing-24-25