University of Tlemcen
Faculty of Sciences
Computer Science Department

Academic Year 2024-2025

L1 Ing Info — ALGEBRA 1

Tutorial sheet 1 : Logic and reasoning

Exercise 1 :(The exclusive OR (XOR))
For two propositions A and B, we define the exclusive OR (XOR) denoted by $A \oplus B$. The statement $A \oplus B$ is true if and only if exactly one of the statements is true.
1- Show that

$$A \oplus B \equiv (A \wedge \bar{B}) \vee (\bar{A} \wedge B) \equiv (A \vee B) \wedge (\bar{A} \vee \bar{B})$$

2- Show that $\oplus$ is commutative.
3- Prove that

$$A \oplus F \equiv A \text{ and } A \oplus A \equiv F,$$

for any false proposition F.
Exercise 2 : Let p and q be two propositions. By using the truth table, prove that

$$(1)\overline{(p \Rightarrow q)} \Leftrightarrow (p \wedge \overline{q}),$$

$$(2)\,(p \Rightarrow q) \Leftrightarrow (\overline{q} \Rightarrow \overline{p}).$$

Exercise 3 : Let A, B, C, D be propositions. Show that :
(A or B) and (C or D) is equivalent to (A and C) or (A and D) or (B and C) or (B and D).
Application : Find pairs of real numbers (x, y) such that :

$$\begin{cases} (x-1)(y-2) = 0 \\ (x-2)(y-3) = 0 \end{cases}$$

Exercise 4 : Give the negation of the following propositions :

1. $[(p \Rightarrow q) \vee r] \wedge (p \vee q)$.
2. $[(p \wedge q) \vee r] \Rightarrow (p \wedge r)$.

Exercise 5 : Let the following mathematical propositions :
$(a) \exists x \in \mathbb{R}, \forall y \in \mathbb{R} : x + y > 0$; $(b) \forall x \in \mathbb{R}, \exists y \in \mathbb{R} : x + y > 0$;
$(c) \forall x \in \mathbb{R}, \forall y \in \mathbb{R} : x + y > 0$; $(d) \exists x \in \mathbb{R}, \forall y \in \mathbb{R} : y^2 > x$.
Are the statements $(a), (b), (c), (d)$ true or false ? Give their negation.
Exercise 6 : Let F the set of women. We note $P(x, y)$ the expression

" x is the daughter of y ", where x and y belongs to F.

Write the following sentences using logical quantifiers :
1. Every woman has at least one daughter.
2. There is at least one woman who has at least one daughter.
3. Every woman has at least one mother.
4. There is at least one woman who has no daughter.

Exercise 7 : Show by induction that for all $n \in \mathbb{N}^*$, we have

$$\sum_{k=1}^{n}(-1)^k k = \frac{(-1)^n(2n+1)-1}{4}.$$

Exercise 8 : The aim of this exercise is to demonstrate the following property by contraposition for $n \in \mathbb{N}^*$:

If the integer (n^2-1) is not divisible by 8, then n is an even integer.

1. Write the contrapositive of the previous proposition.
2. Noting that an odd integer n is written as $n = 4k + r$ with $k \in \mathbb{N}$ and $r \in \{1, 3\}$ (justify), prove the contrapositive.

Exercise 9 : Show by contradiction that

$$\forall a, b \geqslant 0 : \frac{a}{1+b} = \frac{b}{1+a} \Rightarrow a = b.$$