Karnaugh map method

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Outline

Adjacent terms

A Karnaugh map is a graphical representation of the logic system. It can be drawn directly from the truth table, minterm or maxterm Boolean expressions and it gives a minimized sum-of-products or a minimized product-of-sums.

Definition

Two boolean terms are logically adjacent if and only if they contain the same variables and differ in the form of only one variable.

Example

The terms $a \cdot b \cdot c$ and $a \cdot \overline{b} \cdot c$ are logically adjacent, we have $a \cdot b \cdot c + a \cdot \overline{b} \cdot c = a \cdot c$

Karnaugh map

A Karnaugh map is composed of a certain number of cells, each of which is reserved for a term (minterm or maxterm) of a logic function. On each map, the combination of the variables are placed in accordance with the order of Gray's encoding such that adjacent terms are in the neighboring cells or in the cells at map ends. Simplification by Karnaugh map becomes difficult when the number of variables exceeds six.

		a	
		0	1
b	0	0	2
	1	1	3

		ab			
		00	01	11	10
c	0	0	2	6	4
	1	1	3	7	5

		ab			
		00	01	11	10
cd	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

Simplification by Karnaugh map

The simplification of a logic function by Karnaugh map is carried out by grouping the adjacent cells that contains 1*s*. The number of cells in a group must be a power of 2, $(2^k; k = 0, 1, 2, 3, \cdots)$. To minimize a boolean expression using Karnaugh map, we proceed as.

1. We form groups of adjacents cells which contain 1. (We must maximize the number of 1s in each group)
2. Each group of 1s of 2^k adjacent cases gives sa product term of $n - k$; logical variables (n is the number of variables) such that the k^{th} variable which changes state will be eliminated.
3. The minimized expression will be the sum of all minimized terms.

Example

We shall simplify the expression $Y = \overline{a} \cdot b \cdot \overline{c} + a \cdot b \cdot \overline{c}$

c \ ab	00	01	11	10
0	0	1	1	0
1	0	0	0	0

We simplify $Y = a \cdot b \cdot c + a \cdot b \cdot \overline{c} + a \cdot \overline{b} \cdot c + a \cdot \overline{b} \cdot \overline{c}$.

c \ ab	00	01	11	10
0	0	0	1	1
1	0	0	1	1

We simplify $Y = \overline{a} \cdot \overline{b} \cdot \overline{c} + a \cdot \overline{b} \cdot \overline{c}$.

c \ ab	00	01	11	10
0	1	0	0	1
1	0	0	0	0

We simplify $Y = \overline{a} \cdot \overline{b} \cdot c + \overline{a} \cdot b \cdot c + a \cdot b \cdot c + a \cdot \overline{b} \cdot c$

c \ ab	00	01	11	10
0	0	0	0	0
1	1	1	1	1

We simplify $Y = \overline{a} \cdot b \cdot \overline{c} \cdot \overline{d} + a \cdot b \cdot \overline{c} \cdot \overline{d} + \overline{a} \cdot b \cdot \overline{c} \cdot d + a \cdot b \cdot \overline{c} \cdot d + a \cdot b \cdot \overline{c} \cdot d + \overline{a} \cdot b \cdot c \cdot d + a \cdot b \cdot c \cdot d + \overline{a} \cdot b \cdot c \cdot \overline{d} + a \cdot b \cdot c \cdot \overline{d}$.

cd \ ab	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

simplification by Karnaugh map

We simpliy $Y = \overline{a} \cdot \overline{b} \cdot c \cdot d + \overline{a} \cdot b \cdot \overline{c} \cdot d + \overline{a} \cdot b \cdot c \cdot d + \overline{a} \cdot b \cdot c \cdot \overline{d} + a \cdot b \cdot \overline{c} \cdot \overline{d} + a \cdot b \cdot \overline{c} \cdot d + a \cdot b \cdot c \cdot d + a \cdot \overline{b} \cdot c \cdot d$.

cd \ ab	00	01	11	10
00	0	0	1	0
01	0	1	1	0
11	1	1	1	1
10	0	1	0	0

Simplification of POS by Karnaugh

The simplification process of a POS (Product of Sums) form is similar to that used for a SOP (Sum of Products), except that 0*s* need to be grouped to produce minimized sum terms.

Example

We define a logical function by its product of sum.
$f(a,b,c) = (\overline{a}+\overline{b}+c)\cdot(\overline{a}+b+c)\cdot(a+\overline{b}+\overline{c})\cdot(\overline{a}+\overline{b}+\overline{c})\cdot(\overline{a}+b+\overline{c})$.

Simplification of product of sum form using Karnaugh-map

c \ ab	00	01	11	10
0	1	1	0	0
1	1	0	0	0

$$f(a,b,c) = \overline{a} \cdot (\overline{b} + \overline{c}).$$

Incomplętly defined functions

When a combination is physically impossible or invalid, the corresponding function's value is probably unknown or indifferent. We call these combinations "don't care" combinations because we are unable to construct an output expression for them. Binary input states $0000, 0001, 0010, 1101, 1110$, and 1111, for example, are represented as "don't care" in EXCESS-3. The "don't care" emphasises in Karnaugh maps are usually filled with symbols as φ or x. These indifferent states are included in the incompletely described functions simplification procedure, the "don't care" states can be included to the groups of 1s or 0s if necessary.

Example

Using the Karnaugh map, simplify the following function

$$f(a,b,c,d) = \sum(1,2,5,6,9) + \sum \varphi(10,11,12,13,14,15).$$

cd \ ab	00	01	11	10
00	0	0	x	0
01	1	1	x	1
11	0	0	x	x
10	1	1	x	x

We treate an example. Let us define a boolean function f defined by the following truth table.

a	b	c	$f(a,b,c,d$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

The algebric expression is

$$f(a,b,c) = \overline{a}\cdot b\cdot \overline{c} + a\cdot \overline{b}\cdot c + a\cdot b\cdot \overline{c} + a\cdot b\cdot c.$$

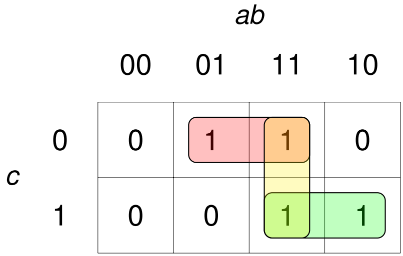
1. Algebric simplification:
   $f(a,b,c) = \overline{a}\cdot b\cdot \overline{c} + a\cdot \overline{b}\cdot c + a\cdot b\cdot \overline{c} + a\cdot b\cdot c =$
   $(\overline{a}+a)\cdot b\cdot \overline{c} + a\cdot c(b+\overline{b}) = b\cdot \overline{c} + a\cdot c.$
2. Karnaugh map's simplification:

Redundant terms

		ab			
		00	01	11	10
c	0	0	1	1	0
	1	0	0	1	1

$$f(a, b, c) = b \cdot \overline{c} + a \cdot c$$

Redundant terms



$$f(a, b, c) = a \cdot b + b \cdot \overline{c} + a \cdot c$$

The first expression is the simplest, we can obtain it after applying the Consensus theorem to the second expression.

Definition

A logical term is considered to be redundant if it covers all of the cells in a Karnaugh map that another term already covers. Without modifying the truth table, this term can be deleted from the equation.