

## 1<sup>ST</sup> YEAR LMD-MATH AND MI

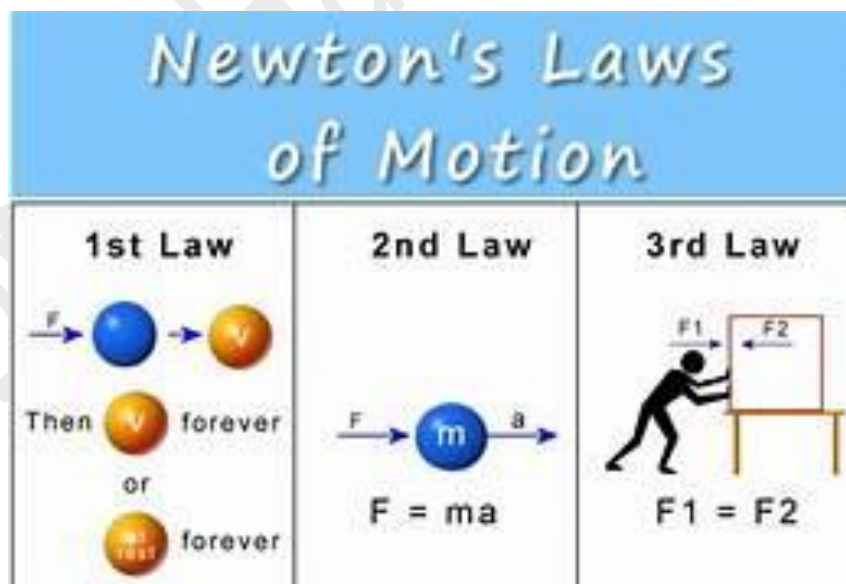
### COURSE OF MECHANICS

### OF THE MATERIAL POINT

## *Chapter V: DYNAMICS OF A PARTICLE*

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### 1. Introduction

In physics, dynamics is the science that studies the relationship between a body in motion and the causes of that motion. It also predicts the motion of a body located in a given environment. Dynamics, more precisely, is the analysis of the relationship between applied force and changes in body motion.

### 2. Newton's laws of motion

#### 2.1. The momentum كمية الحركة

The momentum of a particle is the product of its mass and its instantaneous velocity vector.

$$\vec{P} = m\vec{v}$$

Experiments have shown that the momentum of a system composed of two particles, subject only to their mutual influences, remains constant.

**Theorem:** "In an isolated system of two particles, the variation in the momentum of one particle over time is equal to and opposite in direction to the variation in the momentum of the other particle over the same time".

#### 2.2. Newton's three laws

##### 2.2.1. Galilean principle of inertia مبدأ العطالة (Newton's first laws)

Newton's first law, also known as the law of inertia, states that any object continues to move at a constant speed in a straight line, or remains at rest, unless an external force is applied to it. In other words, if the material body is not subjected to any force, it is either in uniform rectilinear motion, or at rest, if it was initially at rest.

For a particle the principle of inertia thus states: "A free and isolated particle moves in rectilinear motion with constant velocity".

**Note:** A free particle always moves with a constant momentum (principle of inertia).

##### 2.2.2. Newton's second law (Fundamental Principle of Dynamics) المبدأ الأساسي

##### للتحرك

In an abstract sense, force represents the effort required to modify a body's state of motion, in particular to modify its speed. Different bodies have different inertia, i.e. different resistance to a change in their state of inertia, and therefore different resistance to a change in their state of motion. This property must therefore be taken into account in the definition of force. To this end, we introduce a new physical quantity called the momentum of a body.

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$\vec{P} = m\vec{v}$ . Consequently, force can be defined by the derivative of momentum P. This means that the resultant of the forces applied to a particle is:

$$\vec{F} = \frac{d\vec{p}}{dt}$$

This equation is called the "equation of motion»

$$\vec{F} = \frac{dm\vec{v}}{dt} \Rightarrow \vec{F} = m \frac{d\vec{v}}{dt} + \vec{v} \frac{dm}{dt} \overset{=0}{=} m \frac{d\vec{v}}{dt}$$

So  $\vec{F} = m\vec{a}$

This is because the mass m of the moving particle is constant (as is often the case in Newtonian mechanics).

In general, Newton's second law for a moving particle can be written as:

$$\sum \vec{F}_{ext} = m\vec{a}$$

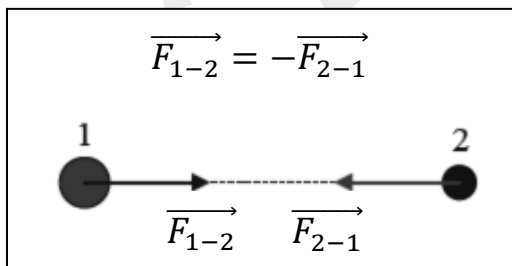
In the S.I. system, the unit of force is: **1Newton=1N=1kg.m.s<sup>-2</sup>**.

**Statement of the Fundamental Principle of Dynamics (2<sup>nd</sup> Newton law) :** in a Galilean frame of reference, the sum of the external forces applied to a system is equal to the derivative of the momentum vector of the system's center of inertia.

### 2.2.3. Newton's third law or principle of action and reaction مبدأ الفعل ورد الفعل

Newton's third law, often referred to as the law of action and reaction, states that for every action, there is an equal and opposite reaction. In other words, when two particles are under mutual influence, the force applied by the first particle on the second is equal to, and opposite in sign to, the force applied by the second particle on the first.

This is shown in the following figure, which allows us to write:



$$|\vec{F}_{1-2}| = |\vec{F}_{2-1}|$$

### 3. Notion of force and law of force

The definition of force by the equation  $\vec{F} = m\vec{a}$  allows us to express the force corresponding to the effect studied as a function of physical factors such as distance, mass, electric charge of the bodies....We will ultimately arrive at deriving "the law of force".

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This law clearly shows the expression of the force (the resultant) applied to a material point in a well-defined situation.

### 3.1. Force of gravity "weight $\vec{p}$ " الثقل او قوة الجاذبية

It's gravitation that makes all the bodies in the universe attract each other. It's an attractive, long-range, low-amplitude force. The gravitational phenomenon is created by the interaction between two bodies. The force of gravity acting on a human being when on Earth is the result of the interaction between the Earth and the human body. As the Earth is more imposing, the gravitational force pulls the human body towards the center of the Earth. This is gravity.

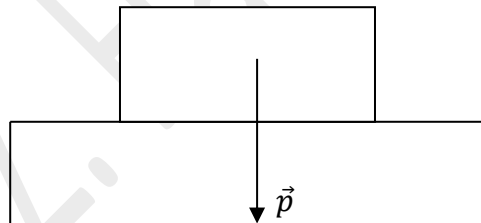
Mass ( $m$ ) is the total amount of matter that makes up an object, while weight ( $p$ ) is the result of the force of gravity ( $g$ ) on mass. The mathematical formula is as follows :

$$p = m \times g.$$

The gravity field is represented at any point on the globe by the vector:  $\vec{p} = m\vec{g}$ .

With  $\vec{g}$  is the gravity acceleration vector, it depends on the altitude and latitude at which the body is located. It is generally considered to be constant, and the value adopted, at mean sea level, is  $9.81 \text{ m.s}^{-2}$ .

Representation of the force of weight:  $\vec{p}$  is always vertical, and directed downwards.



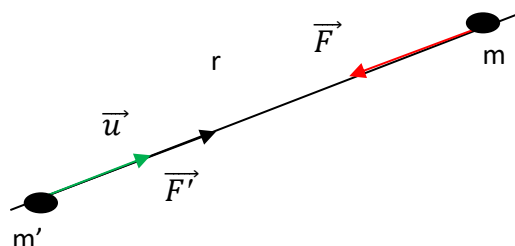
### 3.2. Force at a distance

Assume two bodies separated by a distance  $r$ , of mass  $m$  and  $m'$  respectively.

The attractive force exerted by  $m$  on  $m'$  is :  $\vec{F} = \overrightarrow{F_{m/m'}} = -G \frac{mm'}{r^2} \vec{u}$

The attractive force exerted by  $m'$  on  $m$  is :  $\vec{F}' = \overrightarrow{F_{m'/m}} = G \frac{mm'}{r^2} \vec{u}$

Then:  $\vec{F} = -\vec{F}'$ .



where  $G$  is a constant, the value of which is experimentally determined to be:

$$G = 6.673 \times 10^{-11} \text{ N.m}^2.\text{kg}^{-2}.$$

### 3.3. Force électrique القوة الكهربائية

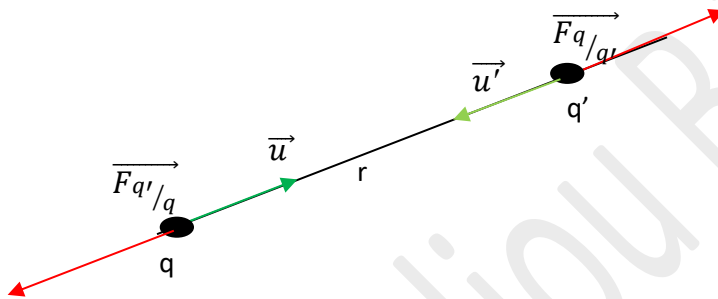
Consider two electric charges  $q$  and  $q'$  separated by a distance  $r$ . The electric force exerted by  $q$  on  $q'$  is given by:

$$\vec{F}_{q/q'} = k \frac{qq'}{r^2} \vec{u}$$

With :  $k$  a constant

The electric force exerted by  $q'$  on  $q$  is given by :

$$\vec{F}_{q'/q} = k \frac{qq'}{r^2} \vec{u}'$$

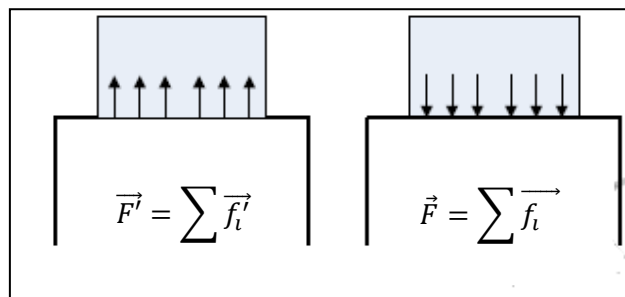


### 3.4. Binding or contact forces القوة رد الفعل

These are the forces acting mutually between bodies in contact.

Consider a solid body placed on a table. The body is in equilibrium on the table, i.e. the acceleration is zero ( $\vec{a} = \vec{0}$ ).

Faced with the force  $\vec{F}$ , representing the resultant of all the interactions of the molecules making up the body, and applied to the table, the latter in turn applies the force  $\vec{F}'$  which is the resultant of all the interactions of the molecules making up the surface of the table that is in contact with the body. The two forces  $\vec{F}$  and  $\vec{F}'$  are called contact or binding forces because of the contact between the two bodies. With  $\vec{F} = -\vec{F}'$  and  $|\vec{F}| = |\vec{F}'|$ .



### 3.5. Friction forces القوة الاحتكاك

Whenever there is contact between the rough surfaces of two solid bodies, a resistance arises that opposes the relative movement of the two bodies. This resistance is called frictional force. Friction is influenced by a number of factors. Consider the type of surface in contact. Smooth surfaces generally offer less friction than rough ones. Friction between solid bodies can be both static and dynamic.

#### a- Static friction force القوة الاحتكاك في حالة السكون

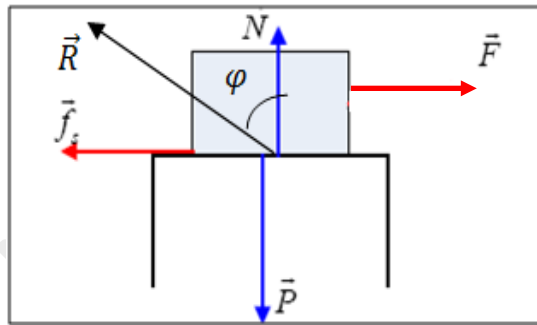
Static friction is the force that keeps a body at rest even in the presence of an external force.

**Example:** A body resting on a horizontal plane :

Consider the body shown in the figure below. It is subjected to four forces.

Let  $f_s$  , be the static friction force and  $\vec{P}$  and  $\vec{N}$  be the weight and normal reaction force of the support respectively.

For the body on the table to move, a minimum force  $\vec{F}$  must be applied.



The mass remains stationary as long as  $F < f_s$ , there is resistance to movement.

In this case the reaction of the support is the resultant force given by :  $\vec{R} = \vec{N} + \vec{f}_s$

At equilibrium:

$$\sum \vec{F}_{ext} = \vec{0} \Rightarrow \vec{N} + \vec{f}_s + \vec{P} + \vec{F} = \vec{0} \Rightarrow \vec{R} + \vec{P} + \vec{F} = \vec{0}$$

By projecting onto the two axes Ox and Oy :

On Oy :  $P=N$  et sur Ox :  $F=f_s$

The mass starts moving when  $F > f_s$

Experience shows that the ratio  $(f_s/N)$  is constant.

$$\operatorname{tg} \varphi = \frac{f}{N} = k = \mu$$

$\mu$ : is the coefficient of friction and  $\varphi$  is the angle of friction.

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The coefficient of friction is called static when the body is stationary. The coefficient of static friction is a ratio between the static frictional force of an object and the normal force, and is written as follows :

$$\operatorname{tg} \varphi = \frac{f_s}{N} = \mu_s$$

### b- Dynamic friction force القوة الاحتكاك في حالة الحركة

Kinetic or dynamic friction is the frictional force present when an object is in motion on another object.

The dynamic friction coefficient is a ratio between the dynamic friction force of an object and the normal force.

Mass starts moving when  $F > f_d$ .

The coefficient of dynamic friction is written as :

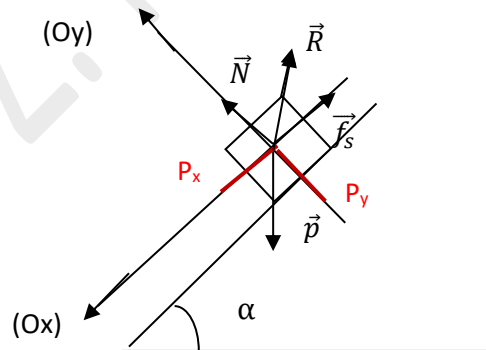
$$\operatorname{tg} \varphi = \frac{f_d}{N} = \mu_d$$

### Application exercise:

Consider a small block of mass  $m$  abandoned without initial velocity at point **A** of an inclined plane at an angle  $\alpha = 30^\circ$  to the horizontal. Point **A** is at height **h**.

1- What is the value of the coefficient of static friction  $\mu_s$  that keeps the mass in equilibrium at point **A**.

Corrected :



What is the value of the coefficient of static friction  $\mu_s$  that keeps the mass in equilibrium at point **A**?



- At equilibrium:

$$\sum \vec{F}_{ext} = \vec{0} \Rightarrow \vec{N} + \vec{f}_s + \vec{P} = \vec{0} \Rightarrow \vec{R} + \vec{P} = \vec{0}$$

Following (Ox)  $-f_s + p_x = 0 \Rightarrow f_s = m g \sin \alpha$

Following (Oy)  $N - p_y = 0 \Rightarrow N = m g \cos \alpha$

In order for the body to remain stationary on the plane, the following conditions must be met  $f_s > p_x$ .

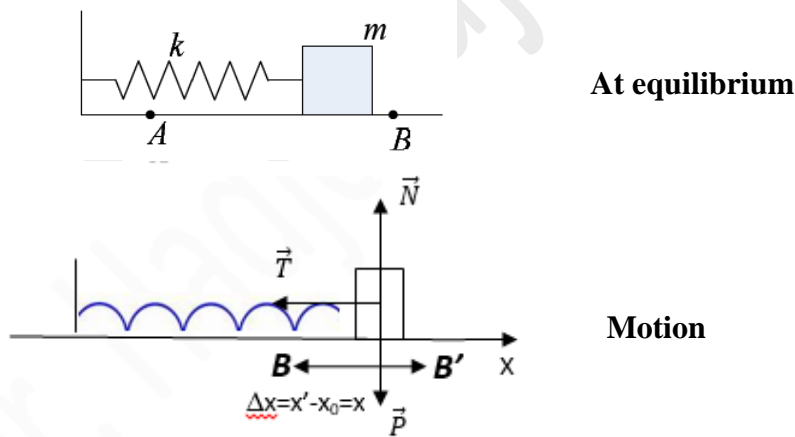
$$\text{We have } \text{tg} \varphi = \frac{f_s}{N} = \mu_s = \frac{m g \sin \alpha}{m g \cos \alpha} = \text{tg} \alpha$$

The maximum value that the coefficient of static friction  $\mu_s$  can take is  $\text{tg} \alpha$ .

**Note:** experience shows that  $\mu_s \geq \mu_d$

### 3.6. Elastic forces قوة الارجاع او قوة المرونية

Elastic force is the force applied to an object that tends to return to its shape after being deformed. Elastic forces cause periodic movements. The most common is sinusoidal motion, as in the case of a spring.



### Elastic force: Spring tension

We have: *FPD* :  $\sum \vec{F}_{ext} = m\vec{a}$

$$\vec{N} + \vec{P} + \vec{T} = m\vec{a}$$

M is the mass of the body

N is the reaction force of the support قوة رد الفعل

P is the force of the body weight قوة الثقل

T is the spring return force قوة الارجاع

By projection :

On Ox we have:  $T = -kx = ma$

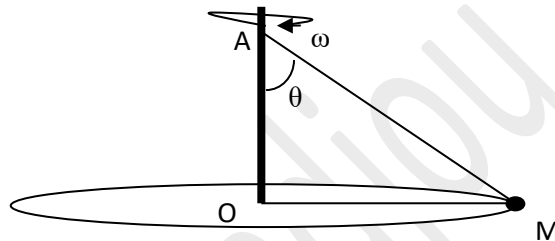
On Oy we have :  $N - P = 0$  So  $N = P = mg$

Where  $k$  is the spring stiffness constant ثابت المرونة .

**Application exercise :**

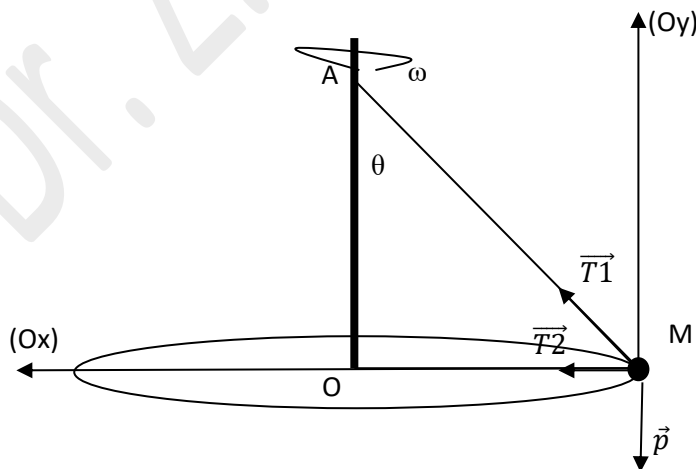
A ball of mass  $m$  is attached by two wires (Am and Om) to a vertical pole. The whole system rotates with a constant angular velocity  $\omega$  around the axis of the post (we know  $g$  the acceleration of gravity,  $\theta$  and  $L = |\overline{OM}|$ )

1. Assuming  $\omega$  is large enough to keep both wires taut, find the force (wire tension) each wire exerts on the ball.
2. What is the minimum angular velocity  $\omega_{\min}$  for which the bottom wire remains taut?



**Corrected :**

Calculating the tension T on the wire:



- 1- Let's find the force (thread tension) that each thread exerts on the ball.

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According to the fundamental principle of dynamics FPD

$$\Sigma \vec{F} = m\vec{a} \Rightarrow \vec{p} + \vec{T}_1 + \vec{T}_2 = m\vec{a}$$

*The ball's motion is circular, so the acceleration in this case is the normal acceleration  $a_N$ , which is directed towards the center of the circle. (with  $a_N = v^2/R$ )*

We choose the reference frame such that :

(Ox) follows the normal acceleration and is directed towards the center of the circle.

By projection onto the axes (Oy) and (Ox) we have :

$$\text{On (Ox) : } T_2 + T_1 \sin \theta = m \cdot a_N \Rightarrow T_2 + T_1 \sin \theta = m \frac{v^2}{R}$$

$$\text{On (Oy) : } p - T_1 \cos \theta = 0 \Rightarrow mg = T_1 \cos \theta$$

$$\text{So } T_1 = \frac{mg}{\cos \theta}$$

$$T_2 = m \frac{v^2}{R} - T_1 \sin \theta = m \frac{v^2}{R} - \frac{mg}{\cos \theta} \sin \theta$$

$$\text{So } T_2 = m \frac{v^2}{R} - m g \tan \theta$$

2- The minimum angular speed  $\omega_{\min}$  at which the bottom wire remains taut.

In order for the lower wire to remain taut, the following conditions must be met  $T_2 \geq 0$

$$T_2 = m \frac{v^2}{R} - m g \tan \theta \geq 0 \Rightarrow \frac{v^2}{R} \geq g \tan \theta$$

$$\text{With } v = \omega R \Rightarrow \frac{\omega^2 R^2}{R} \geq g \tan \theta \quad \text{and } R = OM = L$$

$$\text{So } \omega^2 L \geq g \tan \theta \Rightarrow \omega^2 \geq \frac{g \tan \theta}{L}$$

$$\text{and } \omega \geq \sqrt{\frac{g \tan \theta}{L}} \quad \text{Then } \omega_{\min} = \sqrt{\frac{g \tan \theta}{L}}$$

### References :

1. C.J. Papachristou, Foundations of Newtonian Dynamics: An Axiomatic Approach for the Thinking Student, Nausivios Chora, Vol. 4 (2012) 1531.
2. C.J. Papachristou, Introduction to Electromagnetic Theory and the Physics of Conducting Solids (Springer, 2020).

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