L1-ST A.Y: 2023-2024

Series of Tutorials number 0 - Math2

Exercise 1

Among the following sets, identify those that are vector subspaces:

1)$E_1 = \{(x, y, z, t) \in \mathbb{R}^4 | x = t \text{ et } y = z\}$ 2)$E_2 = \{(x, y, z) \in \mathbb{R}^3 | z = 1\}$
3)$E_3 = \{(x, y) \in \mathbb{R}^2 | x^2 + xy \geq 0\}$ 4)$E_4 = \{(x, y, z) \in \mathbb{R}^3 | x + y + z = 0\}$

Exercise 2

Are the following families linearly independent?
i. $u_1 = (1, 0, 1), u_2 = (0, 2, 2)$ and $u_3 = (3, 7, 1)$ in $\mathbb{R}^3$.
ii. $v_1 = (1, 0, 0), v_2 = (0, 1, 1)$ and $v_3 = (1, 1, 1)$ in $\mathbb{R}^3$.

Exercise 3

In $\mathbb{R}^3$, consider the following subset:

$$E_1 = \{(a + b, b - 3a, a) \in \mathbb{R}^3 | a, b \in \mathbb{R}\}.$$

1. Show that E_1 is a subspace of $\mathbb{R}^3$.
2. Determine a basis B_1 for E_1.
3. Consequently, deduce the dimension of E_1, denoted as $\dim E_1$.

Exercise 4 (SUPP)

Let f be a function defined by:

$$\begin{aligned} f : \quad \mathbb{R}^3 &\longrightarrow \mathbb{R}^2 \\ (x, y, z) &\longrightarrow f(x, y, z) = (2y - 2z, x + y - 2z). \end{aligned}$$

1. Show that f is a linear function.
2. Determine $Kerf$, the kernel of f, then provide a basis for $Kerf$ and deduce the dimension of $Kerf$.
3. Is f injective?
4. Give $dim(Imf)$.
5. Is f surjective?