L1-ST A.Y: 2023-2024

Tutorial Series Number 3 - Math1

Exercise 1

Calculate the limits of the following functions:

$$1)\ \lim_{x\to 1^+}\left(\frac{1}{1-x}-\frac{1}{1-x^2}\right),\qquad 2)\ \lim_{x\to 1}\frac{x^n-1}{x-1}, n\in\mathbb{N}^*\qquad 3)\ \lim_{x\to+\infty}\frac{x-\sqrt{x}}{\ln(x)+x}$$

$$4)\ \lim_{x\to 0}\frac{\ln(1+x^2)}{\sin^2(x)},\qquad 5)\ \lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\ \textbf{(SUPP)},\qquad 6)\ \lim_{x\to+\infty}\sqrt{x+\sqrt{x}}-\sqrt{x}\ \textbf{(SUPP)}$$

Exercise 2

Study the continuity of the following functions:

$$1) f(x)=\begin{cases} x+\dfrac{\sqrt{x^2}}{x} & \text{if } x\neq 0,\\ 0 & \text{if } x=0.\end{cases}\qquad 2)\textbf{(SUPP)} f(x)=\begin{cases} \dfrac{\sin(x)}{|x|} & \text{if } x\neq 0,\\ 1 & \text{if } x=0.\end{cases}$$

Exercise 3

Determine if the following functions can be extended by continuity in $x_0=0$.

$$1) f(x)=\frac{e^x-1}{\sqrt{|x|}},\qquad 2)\textbf{(SUPP)} f(x)=\begin{cases} \dfrac{1-\cos(x)}{x^2} & \text{if } x>0,\\ \dfrac{\sin(x)}{x} & \text{if } x<0.\end{cases}$$

Exercise 4

Study the differentiability on $\mathbb{R}$ of the following functions:

$$1) f(x)=x|x|,\qquad 2)\textbf{(SUPP)} f(x)=\begin{cases} \dfrac{|x|\sqrt{x^2-2x+1}}{x-1} & \text{if } x\neq 1,\\ 1 & \text{if } x=1.\end{cases}$$

Exercise 5

1) Using the mean value theorem, show that

$$\frac{1}{1+x}<\ln(1+x)-\ln(x)<\frac{1}{x}.$$

2) Calculate

$$\lim_{x \to +\infty} x(\ln(1+x) - \ln(x))$$

and deduce from it

$$\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^x.$$

Exercise 6(SUPP)

Let the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} \frac{3 - x^2}{2} & \text{if } \ x < 1, \\ \frac{1}{x} & \text{if } x \geq 1. \end{cases}$$

1) Show that f is continuous on $\mathbb{R}$.

2) Show that f is differentiable on $\mathbb{R}$.

3) By applying the mean value theorem, show that there exists $c \in]0, 2[$ such that

$$2f'(c) = f(2) - f(0).$$