L1-ST A.Y: 2023-2024

Tutorial Series Number 2 - Math1

Exercise 1

In each of the following questions, we are given a set E and subsets A and B of E. Determine explicitly the sets $A \cap B, A \cup B, C_E(B)$, and $C_E(A) \cap B$.

1. $E = \{1, 2, 3, 4\}, \quad A = \{1, 2\}, \quad B = \{2, 4\}$.
2. $E = \mathbb{R}, \quad A =]-\infty, 2], \quad B = [3, +\infty[$.

Exercise 2

Let A be a set, and X, Y, and Z be subsets of A. Prove the following properties:

1. $C_E((X \cup Y)) = C_E(X) \cap C_E(Y)$
2. $X \subset Y \Leftrightarrow C_E(Y) \subset C_E(X)$

Exercise 3:

Let $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$.
Find the following sets:
$f([0, 1[), \quad f(\mathbb{R}), \quad f(]-1, 2[), \quad f^{-1}([1, 2[), \quad f^{-1}([-1, 1]), \quad f^{-1}(\{3\})$.

Exercise 4

Are the following functions injective, surjective, or bijective?

1. f from $\mathbb{R}$ to $[0, +\infty[$ defined by $f(x) = x^2$.
2. g from $[0, +\infty[$ to $[0, +\infty[$ defined by $g(x) = x^2$.

Exercise 5

Let h be the function from $\mathbb{R}$ to $\mathbb{R}$ defined by $h(x) = \frac{4x}{x^2+1}$.

1. Verify that for any nonzero real number a, we have $h(a) = h\left(\frac{1}{a}\right)$. Is the function h injective?
2. Let f be defined on $I = [1, +\infty[$ by $f(x) = h(x)$.
   (a) Show that f is injective.
   (b) Verify that: $\forall x \in I, f(x) \leq 2$.
3. Show that f is a bijection from I to $]0, 2]$ and find f^{-1}.

Exercise 6: (Supp)

Let a, b, c, and d be given non-zero real numbers, and let g be defined as follows:

$$\begin{aligned} g: \quad & \mathbb{R} \setminus \{x_0\} \to \mathbb{R} \setminus \{y_0\} \\ & x \longmapsto g(x) = \frac{ax+b}{cx+d} \end{aligned}$$

1. How should we choose the real number x_0 for g to be a mapping?
2. How should we choose a, b, c, and d for g to be an injective mapping?
3. How should we choose a, b, c, d, and the real number y_0 for g to be a surjective mapping?
4. How should we choose a, b, c, d, x_0, and y_0 for g to be a bijective mapping?