Sets and Mappings

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October 17, 2023

1 Sets

Definition 1.1 *A set is a collection of objects or elements. There is a special case where a set contains no elements, called the empty set, denoted by $\emptyset$.*

Example 1.2 $\{0, 1\}$, $\{\heartsuit, \spadesuit\}$, *set of alphabet letters...*

If x belongs to a set E, then we write $x \in E$. We write $x \notin E$ for the opposite case.

1.1 Inclusion

A set E is included in another set F, denoted as $E \subset F$, if every element of E is also an element of F. In this case, E is a subset of F. If, in addition, the reverse inclusion holds, i.e., $F \subset E$, then we speak of equality in terms of sets. So $E = F \Leftrightarrow (E \subset F)$ and $(F \subset E)$.

1.2 Power Set

Definition 1.3 *Given a set E, we denote $\mathcal{P}(E)$ as the power set of E, defined as:*

$$\mathcal{P}(E) = \{A,\ A \subset E\}$$

Definition 1.4 *The cardinality of a set is the number of elements it contains, denoted as card. For example, if $E = \{a, b, c\}$, then $card(E) = 3$.*

1.3 Complement of a Set

Given a set E and a subset $A \subset E$, the complement of A in E is denoted as $C_E A = \{x \in E : \ x \notin A\}$.

1.4 Union

For two subsets A and B of a set E, the union of A and B is defined as $A \cup B = \{x \in E : \ x \in A \vee x \in B\}$.

1.5 Intersection

For two subsets A and B of a set E, the intersection of A and B is defined as $A \cap B = \{x \in E : \ x \in A \wedge x \in B\}$.

1.6 Set Difference

For two subsets A and B of a set E, the difference of A and B is the set of elements in A that do not belong to B:

$$\begin{aligned} A \setminus B &= \{x \in E : \ x \in A \wedge x \notin B\} \\ &= A \cap C_E B. \end{aligned}$$

1.7 Symmetric Difference

For two subsets A and B of a set E, the symmetric difference of A and B denoted as $A \triangle B$ is defined as:

$$\begin{aligned} A \triangle B &= (A \setminus B) \cup (B \setminus A) \\ &= (A \cup B) \setminus (A \cap B). \end{aligned}$$

Proposition 1.5 *For three subsets A, B, and C of a set E, the following set equality properties hold:*

$C_E(C_E A) = A.$	$A \subset B \Leftrightarrow C_E B \subset C_E A.$
$A \cap A = A.$	$A \cup A = A.$
$A \cap B = B \cap A.$	$A \cup B = B \cup A.$
$A \cap (B \cap C) = (A \cap B) \cap C.$	$A \cup (B \cup C) = (A \cup B) \cup C$
$C_E(A \cap B) = C_E A \cup C_E B.$	$C_E(A \cup B) = C_E A \cap C_E B.$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$

2 Mappings

Definition 2.1 *A mapping f is a relation between two sets A and B, in which each element of the first set (called the domain) is related to a unique element of the second set (the codomain). The elements of the domain A are called preimages. The elements of the codomain B are called images. So, y is the image of x under the mapping f, denoted as $y = f(x)$. The set A is called the domain of definition of f, and the set B is called the codomain of f. We write $f : A \to B$ to represent that f is a mapping from A to B.*

Example 2.2 *Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$. A mapping $f : A \to B$ can be defined as $f = \{(1, a), (2, b), (3, c)\}$.*

2.1 Mapping injetctive(one to one)

Definition 2.3 *A mapping $f : A \to B$ is said to be injective (or one-to-one) if for all $x_1, x_2 \in A$, if $f(x_1) = f(x_2)$, then $x_1 = x_2$.*

2.2 Mapping Surjective(Onto)

Definition 2.4 *A mapping $f : A \to B$ is said to be surjective (or onto) if for every $y \in B$, there exists at least one $x \in A$ such that $f(x) = y$.*

2.3 Mapping bijective

Definition 2.5 *A mapping $f : A \to B$ is said to be bijective if it is both injective and surjective.*

Example 2.6 *Consider the mapping $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = 2x + 1$. This mapping is bijective because it is both injective and surjective.*

Definition 2.7 *The identity mapping on a set A, denoted as I_A or simply I when the set is clear from context, is a mapping that maps each element of A to itself. Formally, $I_A : A \to A$ is defined as $I_A(x) = x$ for all $x \in A$.*

2.4 Composition of mappings

Given two mappings $f : A \to B$ and $g : B \to C$, we can define their composition, denoted as $g \circ f : A \to C$. The composition is defined as:

$$(g \circ f)(x) = g(f(x))$$

for all $x \in A$. In other words, we first apply f to an element x from A, and then we apply g to the result of $f(x)$.

Remark 2.8 *A function is a specific type of mapping where each element of the domain is associated with a unique element in the codomain. All functions are applications, but not all applications are necessarily functions. Functions are a special case of applications with a one-to-one relationship.*