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Academic year: 2023 - 2024
Module : Algebra 2
L1 Math+M1

Remedial exam

Exercise 1 (7 pts)

Let $E = Span\{(1,1,1)\}$ and $F = \{(x,y,z) \in \mathbb{R}^3 : x + y - z = 0\}$

(1) Show that F is a subspace of $\mathbb{R}^3$ and find its dimension.

(2) Find $E \cap F$

(3) Show that $\mathbb{R}^3 = E \oplus F$.

Exercise 2 (5 pts)

We consider the map F defined by :

$$F : \mathbb{R}_2[X] \to \mathbb{R}_2[X]$$
$$P \mapsto (X+1)P'$$

(1) Show that F is a linear map.

(2) Determine $\ker(F)$, the kernel of F and deduce $r(F)$, the rank of F.

(3) Is the map F injective? Surjective?

Exercise 3 (8 pts)

Let $M = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 5 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 5 \end{pmatrix}$

(1) Determine the matrices $A = {}^tM.M$ and $B = A + I_3$, where ${}^tM$ is the transpose matrix of M and I_3 is the unit (identity) matrix of order 3.

(2) Calculate $\det(B)$.

(3) Calculate $B.C$ and $C.B$.

(4) Solve the following system (S) by two methods : (matrix inversion method and Cramer method)

$$(S)\begin{cases} 2x + y = 3 \\ x + 3y + z = 5 \\ y + 2z = 3 \end{cases}$$

(5) Determine the values of $\lambda \in \mathbb{R}$ so that : $\det(B - \lambda I_3) = 0$.

Good luck