University of Tlemcen
Faculty of Science
Department of Mathematic

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1st Year LMD-M and MI

Supervised work N°2 of Mechanics

Vector analysis

Exercise 1

$\vec{\imath}$, $\vec{\jmath}$ *and* $\vec{k}$ being the unit vectors of the rectangular axes Oxyz, we consider the vectors:

$\overrightarrow{r_1}=\vec{\imath}+3\vec{\jmath}-2\vec{k}$, $\overrightarrow{r_2}=4\vec{\imath}-2\vec{\jmath}+2\vec{k}$ and $\overrightarrow{r_3}=3\vec{\imath}-\vec{\jmath}+2\vec{k}$

1. Show these 3 vectors graphically.
2. Calculate their moduli
3. Calculate products $\overrightarrow{r_1}\cdot\overrightarrow{r_2}$ and $\overrightarrow{r_1}\wedge\overrightarrow{r_2}$.

Exercise 2

We give the three vectors $\overrightarrow{V_1}$(1, 1, 0), $\overrightarrow{V_2}$(0, 1, 0) and $\overrightarrow{V_3}$ (0, 0, 2).

1. Calculate norms $\|\overrightarrow{V_1}\|$, $\|\overrightarrow{V_2}\|$ *and* $\|\overrightarrow{V_3}\|$, deduce the unit vectors $\overrightarrow{v_1}$, $\overrightarrow{v_2}$ and $\overrightarrow{v_3}$ respectively from $\overrightarrow{V_1}$, $\overrightarrow{V_2}$ and de $\overrightarrow{V_3}$.

2. Calculate cos $(\widehat{\overrightarrow{v_1},\overrightarrow{v_2}})$, knowing that the corresponding angle is between 0 and π.

3. Calculate the mixed product $\overrightarrow{v_1}.(\overrightarrow{v_2}\wedge\overrightarrow{v_3})$. What does this product represent?

Exercise 3

Consider in space, referred to the direct orthonormal reference frame $(O,\vec{\imath},\vec{\jmath},\vec{k})$ the points A(2, 0,0), B(2, −2, 0) and C(2, 3, −1).

1. Calculate the vector product $\overrightarrow{OA}\wedge\overrightarrow{OB}$
2. Calculate the area of triangle OAB.
3. Calculate the mixed product $(\overrightarrow{OA},\overrightarrow{OB},\overrightarrow{OC})$, Deduce the volume of the parallelepiped built on the vectors.

Exercise 4

Let be a vector $\vec{U}=(t\vec{\imath}+3\vec{\jmath})/(\sqrt{t^2+9})$

1. Show that $\vec{U}$ is a unit vector?
2. Calculate its derivative with respect to time?

Supplementary exercise:

Let be three vectors $\overrightarrow{A}$, $\vec{B}$ *and* $\vec{C}$, such as; $\vec{A}=-2\vec{\imath}+\vec{\jmath}+3\vec{k}$, $\vec{B}=2\vec{\imath}-\vec{\jmath}+\vec{k}$, $\vec{C}=x\vec{\imath}+1\vec{\jmath}+z\vec{k}$

1- Calculate x and z so that the vector $\vec{C}$ or :

a- Parallel to $\overrightarrow{A}$ b- Parallel to $\overrightarrow{B}$

2- If, $\vec{C}=x\vec{\imath}+y\vec{\jmath}+z\vec{k}$ Calculate x, y and z so that the vector $\vec{C}$ or : Perpendicular to $\overrightarrow{A}$ *and* $\vec{B}$ at the same time.