University of Tlemcen
Faculty of Sciences
Department of Mathematic

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1st Year LMD-M and MI

SW N°3 of Mechanics

Kinematics of a Material Point

Exercise 1

A body moves along the x axis according to the relation $x(t)=2t^3+5t^2+5$.

a- Determine the velocity v(t) and acceleration a(t) at each instant t.
b- Calculate the body's position, velocity and instantaneous acceleration for $t_1=2s$ and $t_2=3s$.
c- Deduce the average velocity and acceleration of the body between $t_1$and t_2.

Exercise 2

The coordinates x and y of a moving point M in the plane (Oxy) vary with time t according to the following relationships: $x=t+1$ and $y=(t^2/2)+2$
Find :

- a- The equation of the trajectory
- b- Velocity and acceleration components and their moduli.
- c- Accelerations: normal a_N and tangential a_T, and deduce the radius of curvature.
- d- Nature of motion.

Exercise 3

A particle moves along a trajectory whose equation is $y=x^2$ so that at each instant $v_x=v_0=cst$.
If $t=0$, $x_0=0$.
Determine :

a- The particle's coordinates x(t) and y(t).

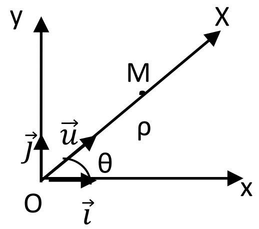
b- The particle's velocity and acceleration.

c- Normal and tangential accelerations and radius of curvature.

Exercise 4

A) A material point M is marked by its cartesian coordinates (x,y).

1. Write x and y in terms of the polar coordinates ρ and θ.
2. Give the expression of the unit vector $\vec{u}$ as a function of the unit vectors $\vec{\imath}$ *and* $\vec{\jmath}$.
3. Calculate $d\vec{u}/d\theta$, what does this vector represent?



B) If the position of point M is given by $\begin{cases} \overrightarrow{OM} = t^2\vec{u} \\ \theta = \omega t \end{cases}$ (ω constant)

Find the expression of the velocity vector $\vec{v}$ in polar coordinates.

Exercise 5

A material point M is identified by its cartesian coordinates (x, y, z).

1. Write down the relationship between cartesian coordinates and cylindrical coordinates (using a diagram).
2. Write the position vector in cylindrical coordinates and deduce the velocity vector in the same coordinate system.
3. If the position of the point is represented in cylindrical coordinates by $\begin{cases} \rho = 4t^2 \\ \theta = \omega t \\ z = \sqrt{t} \end{cases}$

Find the expression of the velocity vector $\vec{v}$ in cylindrical coordinates.

Exercise 6

The motion of a body is defined by the following velocity components:

$$\begin{cases} v_x = R\omega\cos(\omega t) \\ v_y = R\omega\sin(\omega t) \end{cases}$$

Knowing that ω is constant and at t=0, the moving body is at point M (0, R), Determine :

1. The components of the acceleration vector and its modulus.
2. The tangential and normal components of acceleration, and deduce the radius of curvature.
3. The components of the position vector and deduce the equation of the trajectory.
4. What is the nature of the motion?

Exercise 7

A material point M moves along the OX axis with acceleration $\vec{a} = a\,\vec{\imath}$ with a > 0.

1- Determine the velocity vector knowing that v (t=0)= v_0 .

2- Determine the position vector $\overrightarrow{OM}$ given that x(t=0)=x_0.

3- Check that: $v_f^2 \quad - v_i^2 = 2a(x - x_0)$.

4- What is the condition that $\vec{a}.\vec{v}$ so that the motion is uniformly accelerated and retarded?

Exercise 8

The differential of the vector $\vec{r}$, $d\vec{r} = d\vec{l} = dx\vec{\imath} + dy\vec{\jmath} + dz\vec{k}$ can be expressed in cylindrical coordinates as $d\vec{r} = \frac{\partial\vec{r}}{\partial\rho}\,d\rho + \frac{\partial\vec{r}}{\partial\theta}\,d\theta + \frac{\partial\vec{r}}{\partial z}\,dz$.

1. Using the formulas for switching between the two coordinate systems, evaluate the vectors $\frac{\partial\vec{r}}{\partial\rho}$, $\frac{\partial\vec{r}}{\partial\theta}$ et $\frac{\partial\vec{r}}{\partial z}$.
2. Derive the unit vectors $\overrightarrow{U_\rho}$, $\overrightarrow{U_\theta}$ et $\overrightarrow{U_z}$ (cylindrical coordinates) as a function of $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$ (Cartesian coordinates), check that they are orthogonal.
3. Write $\vec{A} = 2x\vec{\imath} + y\vec{\jmath} - 2z\vec{k}$ in cylindrical coordinates.