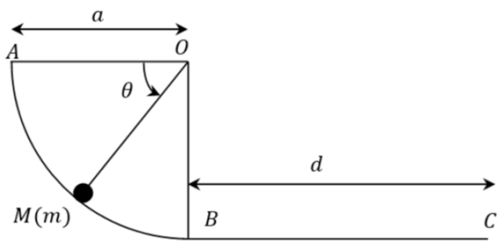
University of Tlemcen
Faculty of Sciences
Department of Mathematic

Academic Year: 2023/2024
1st Year LMD-M and MI

SW n° 06 of Mechanic
Work and Energy

Exercise 1

A particle of mass m, initially at rest in A, slides without friction on the circular surface AOB of radius a.

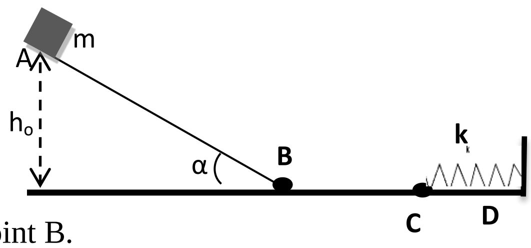


1) Determine the work of weight from A to M.
2) Determine the work of the surface-particle contact force m.
3) Determine the potential energy E_p of m at the point M ($E_p(B) = 0$).
4) Use the kinetic energy theorem to determine the speed of m at point M, deduce its kinetic energy E_c .
5) Calculate the mechanical energy E_m.
6) Show E_c , E_p and E_m ($0 < \theta < \pi 2$). Discuss.
7) The circular surface AOB is connected to a horizontal part BC, there is friction between B and C, the particle stops at a distance d from B. Determine the coefficient of kinetic friction. Given d = 3a = 3m.

Exercise 2

Consider a small block of mass m =5kg dropped without initial velocity at point A of an inclined plane at an angle α=30° to the horizontal. Point A is at a height h_0=5m from the horizontal.

1- Knowing that the coefficient of dynamic friction on plane AB is μ_d=0.2, applying the fundamental principle of dynamics:



- What is the nature of the motion on plane AB?
- Calculate the speed of the block when it reaches point B.

2- After passing through point B at speed V_B, the mass arrives at point C. Knowing that the coefficient of friction is negligible on plane BC :
- Deduce the speed at point C?
- Calculate the maximum compression of the spring, given a stiffness constant equal to k=100N/m? (g =10 m/s^2).

Exercise 3

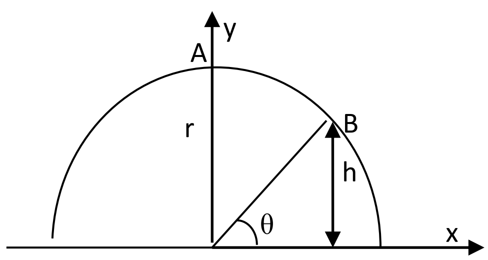
A piece of ice M of mass m slides without friction over the outer surface of an igloo, which is a half-sphere of radius r with a horizontal base.

At t=0, it is released from point A without any initial velocity.

- Find the expression for the velocity at point B, as a function of g, r and θ.
- Using the fundamental relation of dynamics, determine

the expression of $|\vec{N}|$the reaction of the igloo on M at point B as a function of velocity v_B.

- At what height does M leave the sphere?
- At what speed does M arrive at the axis (Ox)?



Corrected exercises

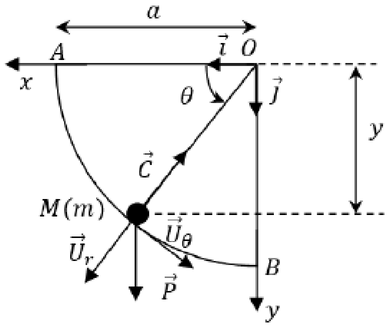
EXERCISE 1

1) The work of $\vec{p}$ from A to M is:

$$dW = \vec{p}.\overrightarrow{dl} \quad \text{with} \quad p = mg\,\vec{j}$$

$$\overrightarrow{dl} = dx\vec{\imath} + dy\vec{\jmath} \quad \text{so} \quad dW = mgdy$$

$$W = mg\int_0^y dy = mgy = mg\ a\ sin\theta$$



2) The work of R_N force is:

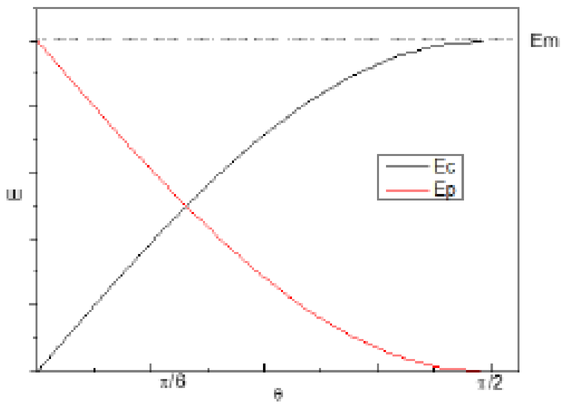
$$W_R = \int_0^y \overrightarrow{R_N}.\overrightarrow{dl} = \vec{0}\ \ Because\ \ \overrightarrow{R_N} \perp \overrightarrow{dl}$$

3) Potential energy:

$dEp = -dW \Rightarrow Ep = -mg\ a.\sin\theta + c$

$E_p\ (B) = 0,\ \theta = \pi/2$ *donc* $c=mga$

$\Rightarrow$ $E_p=mga(1-\sin\theta)$



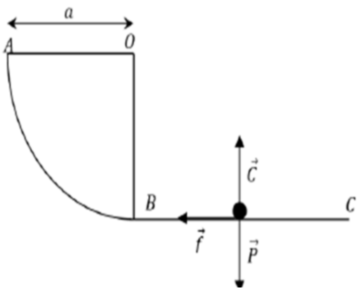
4) $\Delta E_C = \sum W \Rightarrow \frac{1}{2}mv_M^2 = mga\ \sin\theta$

$$V_M=\sqrt{2ga\ sin\theta}$$

5) $E_m = E_c + E_P = mg\ a =$ cste

6) When E_P decreases E_c increases while E_m remains constant.

7) $\mu = \frac{f}{R_N} = \frac{f}{p} \Rightarrow f = \mu mg$



So $\Delta E_C = W_f = \int_B^C \vec{f}.\overrightarrow{dl} = -\mu mgd \ \Rightarrow \frac{1}{2}mv_B^2 = -\mu mgd$

Then $v_B = \sqrt{2ag}$

Note : Replace $\theta = \pi/2$ in the formula for v_M, we find :

$v_B = \sqrt{2ag}$. We cane used also: $E_{m_B} = E_{m_A} \Rightarrow E_{C_A} + E_{P_A} = E_{C_B} + E_{P_B}$

Calculation of μ : we have $\mu = \frac{f}{R_N} = \frac{f}{mg}$ because $R_N = mg$ (with projection on (oy))

$$\sum \vec{F} = m\vec{\gamma} = \vec{f} + \vec{P} + \overrightarrow{R_N}$$

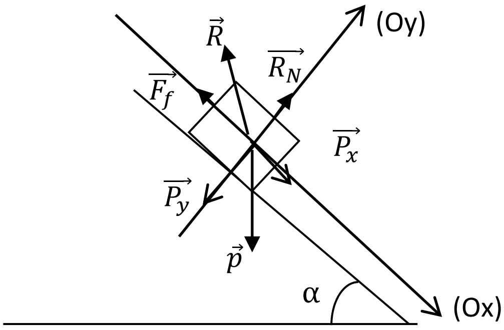
Projection on (ox) : $-f=m.\gamma$

We have also : $v_c^2 - v_B^2 = 2\gamma.d$ ($v_C=0$)

$-v_B^2 = 2\gamma.d = -2ag$ so: $\gamma = \frac{-ag}{d}$ with $-f=m.\gamma=\frac{-mag}{d}$ so $f=\frac{mag}{d}$

Then $\mu = \frac{f}{R} = \frac{f}{mg} = \frac{mag}{mg.d} = \frac{a}{d} = \frac{1}{3}$.

EXERCISE 2



1. Knowing that the coefficient of dynamic friction on plane AB is μ_d=0.2, apply the fundamental principle of dynamics:

- What is the nature of the motion on AB? **a= ?**

$$\Sigma\vec{F} = m\vec{a} = \vec{p} + \overrightarrow{R_N} + \overrightarrow{F_f}$$

Following (Ox) $-F_f + p_x = -F_f + m\,g\,\sin\alpha = ma$

Following (Oy) $R_N - p_y = 0 \Rightarrow R_N = m\,g\,\cos\alpha$

$\mu_d = tg\varphi = F_f / R_N \Rightarrow F_f = R_N\, tg\,\varphi = \mu_d\, m\,g\,\cos\alpha$

$-\mu_d\, m\,g\,\cos\alpha + m\,g\,\sin\alpha = ma \Rightarrow a = g.(\sin\alpha - \mu_d \cos\alpha)$. So: **a = 3.26 m/s²** .

- Calculate the speed of the block when it reaches point B.

$$v_B^2 - v_A^2 = 2al \Rightarrow v_B^2 = 2al = 2a\left(\frac{h}{sin\alpha}\right)$$

$$v_B = \sqrt{2a\left(\frac{h}{sin\alpha}\right)} = 8.074\, m/s$$

2. $V_c = V_B$ because we have an MRU (principle of inertia or Newton's 1st law)

We calculate the compression distance of the spring:

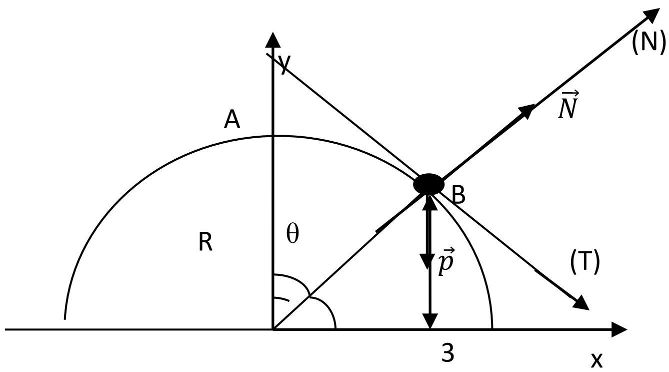
$$\Delta Ec = \Sigma W_{f\,ext} \Rightarrow Ec_D - Ec_C = W_p + W_{Fr} + W_{RN}$$

$$-\frac{1}{2}kx^2 = -\frac{1}{2}mv_c^2 \text{ so : } x = \sqrt{\frac{mv_c^2}{k}} = 1.8\, m$$

2nd Method : Between points C and D

$$E_{M_C} = E_{M_D} \Rightarrow E_{C_C} + E_{P_C} = E_{C_D} + E_{P_D} \Rightarrow \frac{1}{2}kx^2 = \frac{1}{2}mv_c^2 \text{ So; } x = \sqrt{\frac{mv_c^2}{k}} = 1.8\, m$$

EXERCISE 3



1- According to the principle of conservation of mechanical energy between two points A and B:

$$E_{M_A} = E_{M_B} \Rightarrow E_{C_A} + E_{P_A} = E_{C_B} + E_{P_B}$$

So : $E_{C_A} = E_{C_B} + E_{P_B}$

Because $E_{C_A} = 0\ (v_A = O)$ *because the material point is launched without initial velocity*

With $h_B = R cos\theta$

So ; (*)$\Rightarrow$ $mgR = \frac{1}{2} m v_B^2 +$ mg Rcos θ

Then: $gR = \frac{1}{2} v_B^2 +$ g Rcosθ (*)$\Rightarrow$ $v_B^2 = 2(gR - gRcos\theta)$

$$\Rightarrow v_B = \sqrt{2(gR - gRcos\theta)}$$

2- According to the fundamental principle of dynamics :

$$\Sigma \vec{F} = m\vec{a} \Rightarrow \vec{N} + \vec{p} = m\vec{a}$$

We choose a reference frame consisting of the axis (OT) tangent to the half-sphere and the axis (ON) following the radius and in the direction of $\vec{N}$:

Projecting onto (ON) :

N-p cosθ =ma$_N$$\Rightarrow N - mg\cos\theta = -m\frac{v^2}{R}$

3- When point P leaves the sphere N=0 so :

$mg\cos\theta = m\frac{v_p^2}{R} \Rightarrow v_p^2 = Rg\cos\theta$

(*)$\Rightarrow$ $R = \frac{1}{2} R\, g\cos\theta +$ g Rcos $\theta \Rightarrow$ $cos\theta = \frac{2}{3}$ Donc $\theta_0 = 48°$

The material point P leaves the sphere at height: $h_p = \frac{2}{3} R$

The angle relative to the horizontal at which the point leaves the half-sphere is: *90-48=52*

The velocity of the material point at this point:

$v_p^2 = Rg\cos\theta \Rightarrow v_p = \sqrt{\frac{2}{3} Rg}$

4. The velocity of the material point at axis (ox) is:

$v_p^2 = Rg\cos 0 \Rightarrow v_p = \sqrt{Rg}$