Correction of SW N°4 of Mechanics

Material point dynamics

EXERCISE 1:

Modulus of friction force::
M=20kg, $\alpha = 30°$ et F=80N.

1- FPD : $\Sigma\vec{F} = m\vec{a} \Rightarrow \vec{p} + \overrightarrow{R_N} + \vec{F} + \vec{f} = m\vec{a}$ with v=cst so a=0.
Following (Ox): F **cosα- f**=0

Following (Oy): R_N -P =0⇒ R_N =m g

Then : f=80.cos.30=69.28N

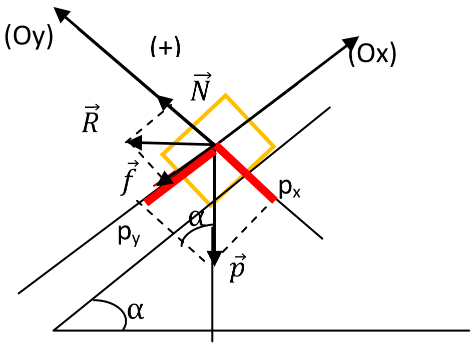
2- Force F for a=1m/s^2

$$\Sigma\vec{F} = m\vec{a} \Rightarrow \vec{p} + \overrightarrow{R_N} + \overrightarrow{F'} + \vec{f} = m\vec{a}$$

Following : (Ox) : -f+ F' **cosα- f**=m.a

$$F' = \frac{m.a + f}{cos\alpha} = 103.1N$$

EXERCISE 2 :



At : t=0 ,v=v_0 and μ=f_d

1- Let's find out how far the block can travel before it stops.

According to the fundamental principle of dynamics:

$$\Sigma\vec{F} = m\vec{a} \Rightarrow \vec{p} + \vec{R} = \vec{p} + \vec{N} + \vec{f} = m\vec{a}$$

Initial velocity v_i= v_0 and final velocity v_f=0 (the body will stop)

We have $v_f^2 - v_i^2 = 2al$ (l being the distance covered by the body)

So ; $a = \frac{v_f^2 - v_i^2}{2l}$

The reference frame must be chosen so that the axis (Ox) follows the axis of motion, so it is parallel to $\vec{f}$ and (Oy) is perpendicular to (Ox), so it is parallel to $\vec{N}$.

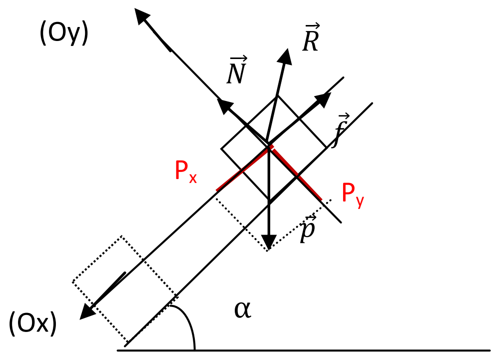
Following (Ox): $-f - p_x = -f - m\,g\,\sin\alpha = ma$

Following (Oy): $N - p_y = 0 \Rightarrow N = m\,g\,\cos\alpha$

$f_d = tg\varphi = f/N \Rightarrow f = N\,tg\,\varphi = N f_d$ so $f = f_d\,m\,g\,\cos\alpha$

$-f_d\,m\,g\,\cos\alpha - m\,g\,\sin\alpha = ma \Rightarrow -f_d\,g\,\cos\alpha - g\,\sin\alpha = \frac{v_f^2 - v_i^2}{2l}$

Then $l = \frac{-v_i^2}{2(-f_d\,g\cos\alpha - g\sin\alpha)} = \frac{v_0^2}{2g(f_d\,\cos\alpha + \sin\alpha)}$



2- The maximum value that the static friction coefficient fs can take for the body to sink,

- At equilibrium

Following (Ox): $-f + p_x = 0 \Rightarrow f = m\,g\,\sin\alpha$

Following (Oy): $N - p_y = 0 \Rightarrow N = m\,g\,\cos\alpha$

For the body to be able to descend, it must: $p_x > f$

$p_x \geq f \Rightarrow m\,g\,\sin\alpha \geq N\,f_s$ (*) ($f_s = f/N$)

with $f = N f_s$ et f_s is the coefficient of static friction at which the body begins its motion (with $f_s = f/N$ $\Rightarrow f = N\,tg\,\varphi$ so $f = f_s\, m\, g \cos\alpha$).

(*)$\Rightarrow m\, g \sin\alpha \geq m\, g \cos\alpha\, f_s$ donc $f_s \leq tg\,\alpha$

The maximum value that can be : f_s is $tg\,\alpha$

3- The velocity v1 of the body as it returns to its initial position;

$x = l$, $v_i = 0$ and we look for v_f

$v_f^2 - v_i^2 = 2al$ l is the distance covered by the body.

So, $a = \frac{v_f^2 - v_i^2}{2l}$

The reference frame must be chosen so that the (Ox) axis follows the axis of motion, i.e. it is parallel to and follows p_x, and the (Oy) axis is perpendicular to (Ox), i.e. it is parallel to $\vec{N}$.

Following (Ox): $-f + p_x = -f + m\, g \sin\alpha = ma$

Following (Oy): $N - p_y = 0 \Rightarrow N = m\, g \cos\alpha$

$f_d = tg\varphi = f/N \Rightarrow f = N\, tg\,\varphi$ so $f = f_d\, m\, g \cos\alpha$

Hence; $-f_d\, m\, g \cos\alpha + m\, g \sin\alpha = ma \Rightarrow -f_d\, g \cos\alpha + g \sin\alpha = \frac{v_f^2 - v_i^2}{2l}$

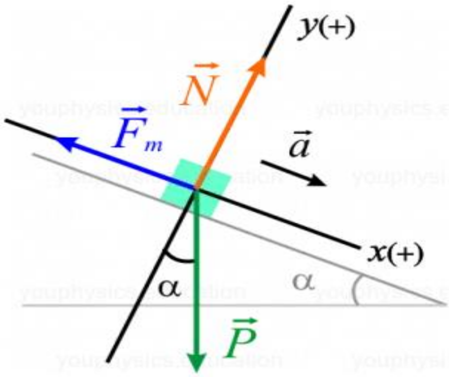
$v_f^2 = 2gl(\sin\alpha - f_d \cos\alpha)$ (where l is the same distance found in the question 1)

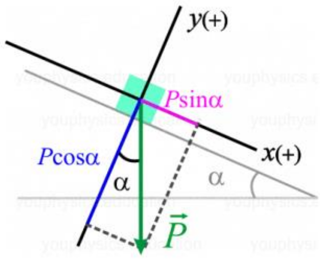
EXERCISE 3

To solve this problem, we'll use Newton's second law. The forces acting on the mass are weight (if it's close to the Earth's surface), the normal (because it's resting on the plane) and the force of the spring, which is given by Hooke's law.

Below is a diagram of the forces acting on the mass and the Cartesian axes we'll use to make the projections.

The spring's restoring force acts in the opposite direction to the spring's elongation (and therefore to the direction of mass displacement):





The acceleration of the mass is also shown in the figure. It runs in the positive direction of the x axis.

In the diagram below, we've plotted the projections of the weight vector on the axes we've chosen.

Newton's second law applied to mass motion gives:

$$\Sigma\vec{F} = m\vec{a} \Rightarrow \vec{p} + \vec{N} + \vec{F_r} = m\vec{a}$$

By projecting onto the Cartesian axes we obtain:

Following (Ox): $- F_r + p_x = - F_r + m\, g \sin\alpha = ma$……..…..1

Following (Oy): $N - p_y = 0 \Rightarrow N = m\, g \cos\alpha$…………………2

We obtain the norm of the support reaction from equation (2), and as you can see, it's not equal to the weight.

N=m g cosα

On the other hand, the norm of the spring return force is given by: $\mathbf{F_r = k.x}$

Finally, solving equation (1) yields the acceleration: $\mathbf{a = (- F_r + m\, g \sin\alpha)/m = 4.8\ m/s^2}$

by taking : $g = 10\ m/s^2$.

<u>EXERCISE 4 :</u>

1- The linear velocity of the body.

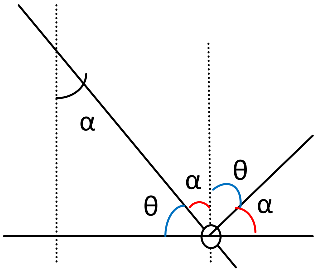
L=30cm , 2α=60°.(α=30°) and ω=10 tr/mn.

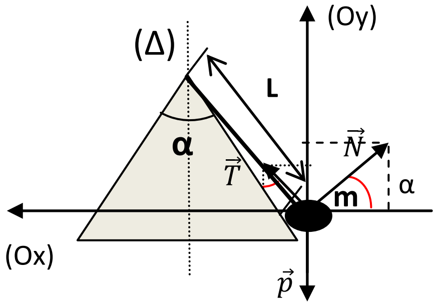
$$\begin{cases} 10x2\pi \longrightarrow 60\ s \\ \omega \longrightarrow 1s \end{cases} \Rightarrow \omega = \frac{10x2\pi}{60} = \frac{\pi}{3} rd/s$$

$v = \omega R$ and $R = l \sin\alpha$

Hence $v = \omega\, l \sin\alpha = \frac{\pi}{3} x0,3xsin\ 30 = 0,157 m/s$

Let's determine the reaction (N) of the surface of the cone on the body and the tension of the wire (T). According to the fundamental principle of dynamics.





α+θ=90

$\Sigma\vec{F} = m\vec{a} \Rightarrow \vec{p} + \vec{N} + \vec{T} = m\overrightarrow{a_N}$

We choose a reference frame such that (Ox) follows the normal acceleration and is directed towards the center of the cone, and axis (Oy) is perpendicular to (N).

Following: (Ox) : $T_x - N_x = m\,a_N$ (*)

Following: (Oy) : $T_y + N_y - p = m\,a_T = 0$ (because $a_T=0$, because the speed is constant)

$\Rightarrow \mathrm{T}\cos\alpha + \mathrm{N}\sin\alpha - \mathrm{p} = 0$

(*)$\Rightarrow$T sin α – N cos α= $m\frac{v^2}{R} = m\frac{\omega^2 R^2}{R}$

So ; $T = \frac{m\omega^2 R}{\sin\alpha} + N\frac{\cos\alpha}{\sin\alpha}$

We replace it in the second equation

$$\left(\frac{m\omega^2 R}{\sin\alpha} + N\frac{\cos\alpha}{\sin\alpha}\right)\cos\alpha + N\sin\alpha - p = 0$$

$$\left(\frac{m\omega^2 R}{\sin\alpha}\cos\alpha + N\frac{\cos^2\alpha}{\sin\alpha}\right) + N\sin\alpha - p = 0$$

$$N\left(\frac{\cos^2\alpha}{\sin\alpha} + \sin\alpha\right) = p - \frac{m\omega^2 R}{\sin\alpha}\cos\alpha \Rightarrow N\left(\frac{1}{\sin\alpha}\right) = \frac{mg\sin\alpha - m\omega^2 R\cos\alpha}{\sin\alpha}$$

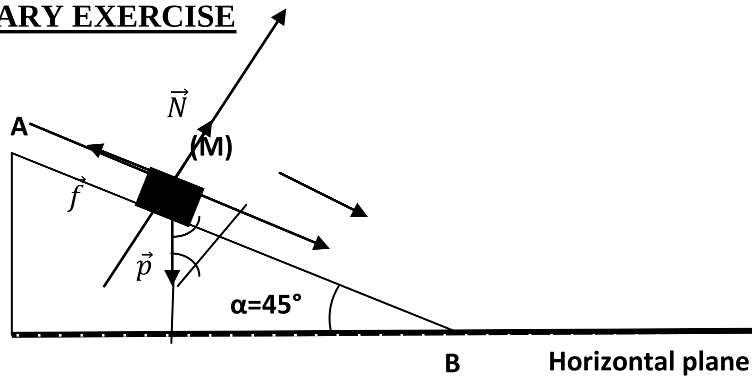
So, $N=m(g\sin\alpha - \omega^2 R\cos\alpha)$, Replacing R with : l sinα , we'll have:

$N=m(g\sin\alpha - \omega^2 l\sin\alpha\cos\alpha) = 7{,}92\ \mathrm{N}$

Hence ; $T = \frac{m\omega^2 l \sin\alpha}{\sin\alpha} + N\frac{\cos\alpha}{\sin\alpha} = 5{,}88$ N

(if we replace N by its expression, we find: T=m g cos α +$m\omega^2 l\,(1-\cos^2\alpha)$ Hence; T= m g cos α +$m\omega^2 l\,\sin^2\alpha$).

SUPPLEMENTARY EXERCISE



v_A=1m/s and μ=0,5 on AB.

The nature of movement on AB: FPD; $\Sigma\vec{F} = m\vec{a} \Rightarrow \vec{p} + \vec{N} + \vec{f} = m\vec{a}$

We choose the reference frame, such that axis (Ox) is along the axis of motion parallel to $\vec{f}$ and (Oy) is perpendicular to (Ox) therefore along $\vec{N}$.

Following: (Ox) $-f + p_x = -f + m\,g\,\sin\alpha = ma$

Following: (Oy) $N - p_y = 0 \Rightarrow N = m\,g\cos\alpha$

μ=tgφ=f/N ⇒f=N tg φ So f= μ m g cosα

Hence - μ m g cosα + m g sinα=ma ⇒ a=g(sinα - μ cosα)

$$a = 10\left(\frac{\sqrt{2}}{2} - 0{,}5\,\frac{\sqrt{2}}{2}\right) = 3{,}54\ m/s^2$$

The acceleration a is constant and positive, so the motion is uniformly accelerated.

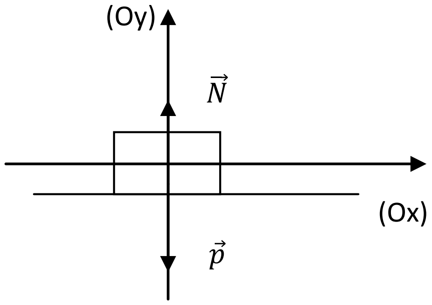
The speed of point M when it reaches point B.

$$v_B^2 - v_A^2 = 2al \Rightarrow v_B^2 = v_i^2 + 2al$$

With ; l=AB=1

$$v_B = \sqrt{1 + 2a} = 2{,}84\ m/s$$

The nature of movement on the horizontal plane: Friction forces are negligible.



$\Sigma\vec{F} = m\vec{a} \Rightarrow \vec{p} + \vec{N} = m\vec{a}$

Following Ox: 0=ma'

Following Oy: N-p=0 $\Rightarrow N = p = mg$

So a'=0 then the motion is uniformly rectilinear.

- Motion is uniform, so speed is constant $v=v_B$ the block will not stop.