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1ST YEAR LMD-M AND MI

COURSE OF MECHANICS

OF THE MATERIAL POINT

Chapter I: Dimensional Analysis and Uncertainty Calculation

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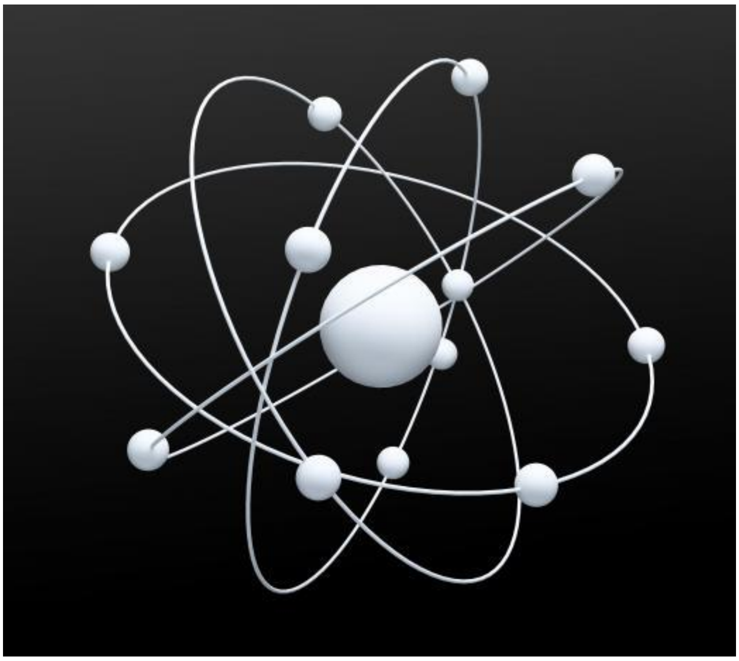

Summary

Part 1: Dimensional analysis

Part 2: Uncertainty Calculation

Part 1: Dimensional analysis

التحليل البعدي

1. Introduction

The observation of physical phenomena is incomplete if it does not lead to quantitative information, which is the measurement of physical quantities. To study a physical phenomenon, one must examine the important variables; the mathematical relationship between these variables constitutes a physical law.

This is possible in certain cases, but for other cases, it is necessary to use a modeling method such as dimensional analysis. *(التحليل البعدي).*

2. Definition of Dimensional Analysis تعريف التحليل البعدي

It is a theoretical tool for interpreting problems based on the dimensions of the involved physical quantities: length, time, mass, and so on.

Dimensional analysis allows for:

- Verifying the validity of dimensioned equations.
- Investigating the nature of physical quantities.
- Exploring the homogeneity of physical laws.
- Determining the unit of a physical quantity based on fundamental units (meter, second, kilogram, etc.).

3. Physical Quantity مقدار فيزيائي

A physical quantity is an observable and measurable property through a specifically designed instrument. Mechanics acknowledges seven fundamental physical quantities: length, time, mass, electric current, temperature, quantity of material, and luminous intensity. Other physical quantities, known as derived quantities, are expressed in terms of these three fundamental quantities, such as velocity, acceleration, force, and more…..

Note :

In general, for first-year students in Mathematics and Computer Science (MI), Mathematics (M), and Computer Science (I), the focus is primarily on the first three fundamental quantities: length, time, and mass.

4. International System of Units الوحدة في النظام العالمي

The value of a physical quantity is given in relation to a standard known as a "unit." The first four fundamental units constitute the MKSA International System (Meter, Kilogram, Second, Ampere). Using these fundamental units, derived units can be constructed: area (m^2), velocity (m/s), force (kg m/s^2)…

Fundamental quantities المقادير الأساسية	Units الوحدة (in the international system MKSA)	Symbolsالرمز
Length	Metre	(m)
Mass	Kilogramme (kg)	(kg)
Time	Seconde	(s)
Current intensity	Ampere	(A)
Temperature	Kelvin	(K)
Light intensity	Candela	(Cd)
Quantity of material	Mole	(mol)

There are specific units such as N (Newton) for force, Hz (Hertz) for frequency, Watt for power, Pascal (Pa) for pressure...

Note: There are two systems of units:

- The International System (SI) known as MKSA (Meter, Kilogram, Second, Ampere), which is the most widely used system.

- The CGS system (Centimeter, Gram, Second), which is less commonly used.

5. Dimensional Equations معادلة ابعاد

Dimension represents the nature of a physical quantity. A physical quantity has only one possible dimension.

The dimension of a quantity G is denoted by : $[\mathbf{G}] = \mathbf{L}$.

By denoting M, L, and T as the dimensions of the fundamental quantities mass, length, and time, we can express the dimensions of other derived quantities in terms of these three. The resulting equations are the dimension equations for these physical quantities.

The Fundamental Quantities المقادير الأساسية	Symbolsالرمز	Dimensions الأبعاد	Units الوحدة (International System (SI))
Length	**L**	$[\mathbf{l}] = \mathbf{L}$	**Mètre (m)**
Mass	**M**	$[\mathbf{m}] = \mathbf{M}$	**Kilogramme (kg)**
Time	**T**	$[\mathbf{t}] = \mathbf{T}$	**Seconde (s)**
Current intensity	**I**	$[\mathbf{I}] = \mathbf{I}$	**Ampère (A)**
Temperature	**T**	$[\mathbf{T}] = \boldsymbol{\theta}$	**Kelvin (K)**
Light intensity	**J**	$[\mathbf{j}] = \mathbf{J}$	**Candela (Cd)**
Quantity of material	**N**	$[\mathbf{n}] = \mathbf{N}$	**Mole (mol)**

Example :

- $[\text{speed}] = [\text{v}] = \frac{[length]}{[\text{time}]} = \frac{[\text{l}]}{[\text{t}]} = \frac{\text{L}}{\text{T}} = \text{LT}^{-1}$ and the unit of speed is m/s
- $[\text{acceleration}] = [\text{a}] = \frac{[\text{speed}]}{[\text{time}]} = \frac{[\text{v}]}{[\text{t}]} = \frac{\text{LT}^{-1}}{\text{T}} = \text{LT}^{-2}$ and the unit of acceleration m/s^2
- $[\text{Force}] = [\text{F}] = [\text{mass}][\text{acceleration}] = [\text{m}][\text{a}] = \text{MLT}^{-2}$ and the unit of force is Newton or (kg.m/s^2).

Notes :

- The dimension of constants is always equal to 1; we say they are dimensionless.
- Angles and functions like sin, cos, tan, exp, ln, and log are dimensionless functions.

[Numeric value] = 1, [angle] = 1, [cos α] = [sin α] = [tan α] = [cot α] = [ln x] = [e^x] = 1

6. Homogeneity of Dimensional Equations تجانس معادلة ابعاد

The two sides of a dimension equation must have the same dimensions since they represent quantities of the same nature.

G is a physical quantity:

$G = A \pm B \Rightarrow [G] = [A] = [B]$

$G = A * B \Rightarrow [G] = [A] * [B]$

$G = A/B \Rightarrow [G] = [A]/[B]$

$G = A^n \Rightarrow [G] = [A]^n$

<u>**Note**</u> :

• A heterogeneous (non-homogeneousغير متجانسة) equation is necessarily False.

• A homogeneous equation is not necessarily true.

• Dimensions cannot be added (or subtracted).

<u>**Example 1**</u>**:** $y = \frac{1}{2}\ at^2 + v_0 t + y_0$ is the equation of a physical law.

Check that this equation is homogeneous?

This equation is homogeneous if:

$$[y] = \left[\frac{1}{2}\ at^2\right] = [v_0 t] = [y_0]$$

We have :

$[y] = [y_0] = L,\ \left[\frac{1}{2}\ at^2\right] = \left[\frac{1}{2}\ \right][a][t]^2 = 1\ LT^{-2}T^2 = L\ ,\ [v_0 t] = [v_0][t] = LT^{-1}T = L$

So :
$[y] = \left[\frac{1}{2}\ at^2\right] = [v_0 t] = [y_0]$ is checked.

Hence the equation $y = \frac{1}{2}\ at^2 + v_0 t + y_0$ is homogeneous.

Notes :

We can use this property of dimension equations to discover physical laws by knowing the variables involved in the given physical phenomenon and the relationship among them.

Example 2:

The period is given in terms of length and severity by the following relationship:

$$T=k\,.l^{x}.g^{y}$$

Give the physical law of period T الطور?

For this it is necessary to determine the exponents x and y.

It is assumed that the equation is homogeneous so: $[\boldsymbol{T}] = [\boldsymbol{k}][\boldsymbol{l}]^{\boldsymbol{x}}[\boldsymbol{g}]^{\boldsymbol{y}}$

The dimensions of all physical quantities in the study relationship are written.

$$[l] = L, [k] = 1, \text{T is a time } [T] = T\ et$$

We have weight force قوة الثقل p=mg with:

$$[\boldsymbol{p}] = [\boldsymbol{m}][\boldsymbol{g}] \Rightarrow [\boldsymbol{g}] = \frac{[\boldsymbol{p}]}{[\boldsymbol{m}]}$$

P : the weight is a force so it has the dimension of a force: $[p] = [F] = MLT^{-2}$

So $[g] = \frac{[F]}{[m]} = \frac{MLT^{-2}}{M} = LT^{-2}$

$$\text{g is the acceleration } [g] = LT^{-2}$$

Hence : $T=1\ .\ L^{x}\ .\ (LT^{-2})^{y} \Rightarrow M^{0}L^{0}T^{1} = L^{x+y}\,.T^{-2y}$

By identification we will have:

$\begin{cases} x + y = 0 \\ -2y = 1 \end{cases} \Rightarrow \begin{cases} y = -\frac{1}{2} \\ x = -y = \frac{1}{2} \end{cases}$ so $T = k\, l^{\frac{1}{2}}\, g^{-\frac{1}{2}} \Rightarrow T = k\sqrt{\frac{l}{g}}$ it's the law of the period.

Example 3:

The average speed of the particles is expressed as a function of the mass m, the volume V, and the pressure p by the fallowing expression:

v = f (m, V, p)= $k.m^{\alpha}.V^{\beta}.p^{\gamma}$

It is assumed that the equation is homogeneous, therefore: $[v] = k[m]^{\alpha}[V]^{\beta}[p]^{\gamma}$ (1)

with $[m] = M, [v] = LT^{-1}, [V] = L^3, [p] = \frac{[F]}{[s]} = \frac{[m][a]}{[s]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

(1)⇒ $M^0LT^{-1} = M^{\alpha}L^{3\beta}(ML^{-1}T^{-2})^{\gamma}$

$$\Rightarrow M^0LT^{-1} = M^{\alpha+\gamma}L^{3\beta-\gamma}T^{-2\gamma}$$

By identification we will have:

$$\begin{cases} \alpha + \gamma = 0 \\ 3\beta - \gamma = 1 \\ -2\gamma = -1 \end{cases} \Rightarrow \begin{cases} \gamma = \frac{1}{2} \\ \alpha = -\gamma = -\frac{1}{2} \\ \beta = \frac{1+\gamma}{3} = \frac{1+1/2}{3} = \frac{1}{2} \end{cases}$$

So : $v = km^{-\frac{1}{2}}V^{\frac{1}{2}}p^{\frac{1}{2}} \Rightarrow v = k\sqrt{\frac{pV}{m}}$ It's a law of the average speed of the particles

Conclusions :

Dimensional analysis serves the following purposes:

- Verification of the homogeneity of physical formulas.

- Determination of the nature of a physical quantity.

- Exploration of the general form of physical laws.

2nd part: Uncertainty Calculation

حساب الارتيابات

1. Introduction

In an experiment, exact measurements do not exist. These are always accompanied by more or less significant errors depending on the measurement method used, the quality of the instruments used, and the role of the operator. The measuring instrument, even if built upon a standard, also has a certain precision as provided by the manufacturer. Therefore, measurements are carried out with approximations. Estimating the errors made in measurements and their consequences is essential.

2. Absolute and relative uncertaintyالارتياب المطلق و الارتياب النسبي

2 .1. Asolute errorالخطا المطلق

The absolute error of a measured quantity G is the difference δG between the experimental value Gm and a reference value that can be considered as exact, Ge. In reality, since the exact value is inaccessible, it is approximated by taking the average of a series of measurements of the quantity G.

$$\boldsymbol{\delta G = |G_{mesurée} - G_{exacte}|}$$

2 .2 Relative errorالخطا النسبي

The relative error is the ratio of the absolute error to the reference value. The relative error is dimensionless; it indicates the quality (precision) of the obtained result. It is expressed in terms of a percentage.

$$\boldsymbol{\frac{\delta G}{G} = \frac{|G_{mesurée} - G_{exacte}|}{G_{mesurée}}}$$

2.3. Absolute uncertaintyالارتياب المطلق

This is the maximum error that can be committed in the evaluation.

$$\Delta G \geq |\delta G| \Rightarrow \Delta G = |G_{ex} - G_m| \;\Rightarrow G_{ex} = G_m \pm \Delta G$$

$$\Rightarrow G_m - \Delta G \leq G_{ex} \leq G_m + \Delta G$$

the general form is :

$$G_{ex} = G_m \pm \Delta G$$

The absolute uncertainty has the same unit as the measured quantity and is always positive.

Example : m=12,121g et Δm=0,02g

The correct expression for the measurement of m is: m=(12,121±0,020) g

2.4. Relative uncertaintyالارتياب النسبي

The relative uncertainty is the ratio between the absolute uncertainty and the measured value of G. It is also expressed in terms of a percentage and is a convenient way to quantify the precision of a measurement. It is denoted as: ΔG/G

It is given in percentage and it is always smaller than 1.

$$\frac{\Delta m}{m} = \frac{0,02}{12,121} \approx 0.16\%$$

3. Uncertainty Calculationحساب الارتيابات

Generally, there are two mathematical methods for uncertainty calculation: the total differential method, which is a general approach, and the logarithmic method, which is limited to physical laws expressed as a product or a ratio.

3.1. The total differential method الطريقة التفاضلية الكلية

Let f(x, y, z) be a function that depends on three variables x, y, z:

The total differential of the function (f) is expressed by the following equation:

$$\boldsymbol{df} = \left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right)_{y,z=cst} \boldsymbol{dx} + \left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}}\right)_{x,z=cst} \boldsymbol{dy} + \left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{z}}\right)_{x,y=cst} \boldsymbol{dz}$$

$\left(\frac{\partial f}{\partial x}\right)$ is the partial differential of the function f with respect to x , considering y and z as constants.

$\left(\frac{\partial f}{\partial y}\right)$ is the partial differential of the function f with respect to y, considering x and z as constants.

$\left(\frac{\partial f}{\partial x}\right)$ is the partial differential of the function f with respect to z , considering x and y as constants.

The absolute uncertainty on (f) is generally expressed in the form:

$$\Delta \boldsymbol{f} = \left|\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right| \Delta \boldsymbol{x} + \left|\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}}\right| \Delta \boldsymbol{y} + \left|\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{z}}\right| \Delta \boldsymbol{z}$$

__Example:__

Let f (x, y) be a physical quantity that depends on two variables x and y.

f is expressed as f (x,y) =2xy +x^2y

The total differential of "f " will be given by : $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$

with $\frac{\partial f}{\partial x} = 2y + 2xy$ et $\frac{\partial f}{\partial y} = 2x + x^2$

so $df = (2y + 2xy)dx + (2x + x^2)dy$

Hence the absolute uncertainty on the quantity « f » is given by:

$$\Delta f = |2y + 2xy|\Delta x + |2x + x^2|\Delta y$$

3.2. Logarithmic methodالطريقة اللوقاريتمية

This method is based on the logarithm and its derivative.

Consider a three-variable function, G = f(x, y, z). To calculate the relative uncertainty on the function G using the logarithmic differential method, the following steps should be followed:

1. Introduce the logarithmic function to the function G.

2. Calculate d(logG) = dG / (G ln 10) or d(lnG) = dG / G.

3. $\frac{dG}{G} \le \frac{\Delta G}{G}$, and deduce the relative uncertainty on G.

Example

Let the function $f(x,y,z) = x^a y^b z^c$

x, y et z are a variables and a, b et c are constants in the exponents.

First, we find the logarithm of "f":

$\log f = \log x^a y^b z^c = \log x^a + \log y^b + \log z^c = a \log x + b \log y + c \log z$

Then, we calculate the derivative of log f:

$$d\log f = a\, d\log x + b\, d\log y + c\, d\log z$$

$$\frac{df}{f} = a\,\frac{dx}{x} + b\,\frac{dy}{y} + c\,\frac{dz}{z}$$

$$\frac{\Delta f}{f} = \left|\frac{a}{x}\right| \Delta x + \left|\frac{b}{y}\right| \Delta y + \left|\frac{c}{z}\right| \Delta z$$

If x, y, z, a, b and c are positive constants, we can write $\Delta f/f$ by :

$$\frac{\Delta f}{f} = a\,\frac{\Delta x}{x} + b\,\frac{\Delta y}{y} + c\,\frac{\Delta z}{z}$$

$\frac{\Delta f}{f}$ Represents relative uncertainty الارتياب النسبي.