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Chapiter 2 : Complex Analysis

1. Write in the algebric form $(a+ib)$ the complex number $\frac{1}{z}$ knowing that $z=\frac{1}{3}+i\frac{4}{5}$.
2. Show that : if $\lambda\in\mathbb{R}$, then $\left|\frac{1+\lambda i}{1-\lambda i}\right|=1$.
3. Solve the equation $(1+i)Z=3+i$.
4. Find all complex numbers $z\in\mathbb{C}^*$ such that $Z=(z+1)\frac{1}{z}$ is a real number.
5. Find all complex numbers $z\in\mathbb{C}$ such that $Z=(\overline{z}-2)(z-i)$ is a purely imaginary number.
6. Find the modulus of the following complex numbers
$$\frac{1+i}{1-i};\quad \frac{(2+3i)(1-5i)}{\left(4+i\sqrt{10}\right)\left(\sqrt{12}-i\right)};\quad a(\cos\theta+i\sin\theta), a\in\mathbb{R}^*;\quad 1+\cos\theta+i\sin\theta.$$
7. (a) Find the square roots of the complex number $(35-12i)$ under them algebric form.
   (b) Solve in $\mathbb{C}$ the equation : $z^2-(10+3i)z+14+18i=0$.
8. Solve in $\mathbb{C}$ the equation : $z^2+(2-2i)z+3-6i=0$.
9. Solve in $\mathbb{C}$ the equations :(a) $z^2=15+8i$. (b) $z^2=24+70i$. **(Additional)**.
10. Calculate the complex square roots of $z=1-i$. Give the results in algebric and trigonometric forms. Deduce then that :
$$\cos\left(\frac{\pi}{8}\right)=\sqrt{\frac{1+\sqrt{2}}{2\sqrt{2}}}\quad\text{and}\quad\sin\left(\frac{\pi}{8}\right)=\sqrt{\frac{1}{2\sqrt{2}\left(1+\sqrt{2}\right)}}.$$
11. Write the following complex numbers in the trigonometric or the exponential form :
$$z_1 = 1+i\sqrt{3};\quad z_2=-1+i\quad ; z_3=1+\cos\theta+i\sin\theta,\quad \theta\in[0,2\pi[.$$
$$z_4 = \frac{\left(1-i\sqrt{3}\right)^2}{(1+i)^3}.$$
12. Simplify the following complex: $z=\frac{(1+i)^9}{(1-i)^7}$.
13. Expand $\cos(3\theta)$ and $\sin(3\theta)$ in a polynomial in terms of $\cos\theta$ and/or $\sin\theta$.
14. Linearize $\cos^2(\theta), \sin^3(\theta), \cos^4(\theta)$.
15. Solve in $\mathbb{C}$ the equation : $z^3=-1$.
16. Calculate the modulus and the argument of : $4\sqrt{2}(1-i)$. Deduce its exponential writting, and the solutions in $\mathbb{C}$ of the equation : $z^3=4\sqrt{2}(1-i)$. Represent them on the complex plan.
17. Solve in $\mathbb{C}$ the equation : $z^3+i=0$ then the equation $z^3+i=i(z-i)$.
18. Write the complex number $(1+i)$ in its exponential form, then find the value of the complex number :
$$z=(1+i)^{12}+(1-i)^{12}.$$