Tlemcen University
Faculty of Sciences
Mathematics Department

Final Exam
Machine Framework 1
S1-1B-Maths/MI

14/01/2024
1h30 mn

Exercise 1 (Cours-6 points).

1. *(1.00 points) If a number has n digits in hexadecimal representation, how many digits (bits) does it have in binary ?*
2. *(1.00 points) Give the biggest binary number that you can write with 6 bits.(Give the binary and decimal value)*
3. *(1.00 points) How many bits do you need to write the number* 612 *in binary ?*
4. *(1.00 points) Give the interval of numbers that you can represent in two's complement with n bits.*
5. *(1.00 points) What is the largest (most positive) and the smallest (most negative) quantity that may be represented with eight bits in two's complement ? Give the decimal representation.*
6. *(1.00 points) Give the expression of the group of* 0*s.*

c \ ab	00	01	11	10
0	1	1	0	0
1	1	1	1	1

Exercise 2 (4 points).

1. *(1.50 points) Give the number of characters in the following $UTF-8$ code.*

$$E2\ A3\ B8\ 38\ 1F.$$

2. *(1.50 points) Convert the IEEE 754 simple precision hexadecimal number $X = BF700000$ to decimal.*
3. *(1.00 points) Write* 145 *in Gray code representation .*

Exercise 3 (5 points). *Let us consider the logic function defined by*

$$f(a,b) = a.b \oplus \overline{b}.\overline{c}.$$

1. *(2.00 points) Prove that* $f(a,b) = a.b + \bar{b}.\bar{c}$.
2. *(3.00 points) Implement a logic circuit using only* 2− *input NAND gates.*

Exercise 4 (5 points)**.** *Let us define the logic function* f *by the following truth table.*

A	B	C	D	$f(A,B,C,D)$
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

1. *(1.75 points) Write the product of sums expression of* f.
2. *(1.50 points) Give the maxterms and minterms expressions of* f. *Deduce the sum of products expression of* f.
3. *(1.75 points) Using the Karnaugh map, simplify the sum of products expression of* f.

Version française

Exercice 1 (Cours-6 points)**.**

1. *(1.00 points) Soit un nombre codé en héxadecimal avec* n *chiffres. Combien de bit (chiffre) faudra t-il pour coder ce même nombre en binaire ?*
2. *(1.00 points) Donner le plus grand nombre binaire codé sur* 6 *bits.*
3. *(1.00 points) Combien de bits faudra t-il pour coder* 612 *en binaire ?*
4. *(1.00 points) Donner l'intervalle des nombres représentables en complément à* 2 *sur* n *bits.*
5. *(1.00 points) Donner la plus grande valeur et la plus petite valeur codée en complément à 2 sur 8 bits. Donner les valeurs en décimal.*
6. *(1.00 points) Donner l'expression du groupe formé de* 0*s.*

c \ ab	00	01	11	10
0	1	1	0	0
1	1	1	1	1

Exercice 2 (4 points)**.**

1. *(1.50 points) Donner le nombre de caractères de l'expression codée en* $UTF-8$ *suivante.*

$$E2\ A3\ B8\ 38\ 1F.$$

2. *(1.50 points) Convertir en décimal le nombre* $X = BF700000$ *; codé en* $IEEE$ 754 *simple précision hexadecimal.*
3. *(1.00 points) Ecrire* 145 *en code de Gray.*

Exercice 3 (5 points)**.** *On considère la fonction logique définie par l'expression algébrique suivante*

$$f(a,b) = a.b \oplus \overline{b}.\overline{c}.$$

1. *(2.00 points) Montrer que* $f(a,b) = a.b + \overline{b}.\overline{c}$*.*
2. *(3.00 points) Tracer le circuit logique qui représente la fonction logique* f *en utilisant que des portes NAND à deux entrées.*

Exercice 4 (5 points). *On définit une fonction logique f définie par sa table de vérité ci dessous..*

A	B	C	D	$f(A, B, C)$
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

1. *(1.75 points) Donner l'expression de f sous forme de produit de sommes.*
2. *(1.50 points) Ecrire l'expression de f en fonction des maxtermes et mintermes. En déduire l'expression de f sous forme de somme de produits.*
3. *(1.75 points) Simplifier l'expression de somme de produits de f, en utilisant le tableau de Karnaugh.*

2

Exercice 1 (Cours-6 points).

1. *(1 point) If a number has n digits in hexadecimal representation, how many digits (bits) does it have in binary ?*

> ***Solution:*** *A number of n digits in hexadecimal will have $4.n$ bits in binary.* **(01)**

2. *(1 point) Give the biggest binary number that you can write with 6 bits.(Give the binary and decimal value)*

> ***Solution:*** *The biggest binary number with 6 bits is given by* $111111_2 = 2^6 - 1 = 63$.**(01)**

3. *(1 point) How many bits do you need to write the number 612 in binary ?*

> ***Solution:*** *We have* $612 \overbrace{=}^{(\mathbf{0.5})} 2^n - 1 \implies 613 = 2^n \implies n = \frac{ln(613)}{ln2} \simeq 9.25$, *then* $n = 10$**(0.5)** *is the number of bits to encode 612 in binary.*

4. *(1 point) Give the interval of numbers that you can represent in two's complement with n bits.*

> ***Solution:*** *We can represent in two's complement with n bits, the numbers from* -2^{n-1} *to* $+(2^{n-1} - 1)$.**(01)**

5. *(1 point) What is the largest (most positive) and the smallest (most negative) quantity that may be represented with eight bits in two's complement ? Give the two's complement and the decimal representation.*

> ***Solution:***
> — *The largest number that we can represent in two's complement with 8 bits is* $01111111_{2C} = +(2^7 - 1) = +127$.**(0.5)**
> — *The smallest number that we can represent in two's complement with 8 bits is* $10000000 = -2^7 = -128$.**(0.5)**

6. *(1 point) Give the expression of the group of* 0*s.*

c \ ab	00	01	11	10
0	1	1	0	0
1	1	1	1	1

Solution: *The value of the zeros group is* $\overline{a} + c$.**(01)**

Exercice 2 (4 points)**.**

1. *(1½ points) Give the number of characters in the following* $UTF-8$ *code.*

$$E2\ A3\ B8\ 38\ 1F.$$

Solution: *We write the binary equivalent of the code.*
$E2\ A3\ B8\ 38\ 1F \overbrace{=}^{(\mathbf{0.5})} 11100010\ 10100011\ 10111000\ 00111000\ 00011111,$
then 11100010 10100011 10111000 *represents one* $UTF-8$ *character***(0.5)**, 00111000 *represents an ASCII character***(0.25)** *and* 00011111 *an other ASCII character***(0.25)**. *Then we have three* (3) *charcters.*

2. *(1½ points) Convert the IEEE 754 simple precision hexadecimal number* $X = BF700000$ *to decimal.*

Solution: $X = BF700000 \overbrace{=}^{(\mathbf{0.25})} 10111111011100000000000000000000_2$, *The MSB* 1 *represents the sign - (negatif number)***(0.25)**, $E_b = 01111110_2 = 126$, *we have* $E_b = E + 127$ *then the real exposant is* $E = -1$**(0.25)**, $M = 111$**(0.25)**. *Then the number is* $X \overbrace{=}^{(\mathbf{0.25})} -1,111.2^{-1} = -0.1111_2 = -(0 + 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4}) = -0.9375$**(0.25)**

3. *(1 point) Write* 145 *in Gray code representation.*

Solution: *First we convert* 145 *to straight binary representation ;* $145 \overbrace{=}^{(\mathbf{0.25})} 10010001_2$, *then we convert to Gray code* $\begin{array}{ccccccccccccccc} 1 & + & 0 & + & 0 & + & 1 & + & 0 & + & 0 & + & 0 & + & 1 \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ 1 & & 1 & & 0 & & 1 & & 1 & & 0 & & 0 & & 1 \end{array}$ *finally,* $145 = 11011001_{Gray\ code}$.$(\mathbf{0.75})$

Exercice 3 (5 points). *Let us consider the logic function defined by*

$$f(a,b) = a.b \oplus \overline{b}.\overline{c}.$$

1. *(2 points) Prove that* $f(a,b) = a.b + \overline{b}.\overline{c}$.

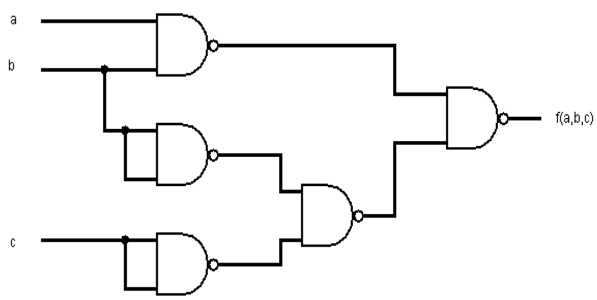
Solution: *By definition of the xor operation, we can write.*

$$\begin{aligned} f(a,b) &\overset{(\mathbf{0.25})}{\overbrace{=}} a.b.\overline{\overline{b}.\overline{c}} + \overline{a.b}.\overline{b}.\overline{c}. \\ &\overset{(\mathbf{0.25})}{\overbrace{=}} a.b.(b+c) + (\overline{a}+\overline{b}).\overline{b}.\overline{c} \\ &\overset{(\mathbf{0.25})}{\overbrace{=}} a.b.b + a.b.c + \overline{a}.\overline{b}.\overline{c} + \overline{b}.\overline{b}.\overline{c} \\ &\overset{(\mathbf{0.5})}{\overbrace{=}} a.b + a.b.c + \overline{a}.\overline{b}.\overline{c} + \overline{b}.\overline{c} \\ &\overset{(\mathbf{0.25})}{\overbrace{=}} a.b(1+c) + (\overline{a}+1).\overline{b}.\overline{c} \\ &\overset{(\mathbf{0.5})}{\overbrace{=}} a.b + \overline{b}.\overline{c} \end{aligned}$$

2. *(3 points) Implement a logic circuit using only* $2-$ *input NAND gates.*

Solution:

$$\begin{aligned} f(a,b) &\overset{(\mathbf{0.25})}{\overbrace{=}} \overline{\overline{a.b + \overline{b}.\overline{c}}} \\ &\overset{(\mathbf{0.25}}{\overbrace{=}} \overline{\overline{a.b}.\overline{\overline{b}.\overline{c}}} \\ &= \overline{\overline{a.b}.\overline{\underbrace{\overline{b.b}}_{(\mathbf{0.5})}.\underbrace{\overline{c.c}}_{(\mathbf{0.5})}}} \end{aligned}$$



(1.5)

Exercice 4 (5 points). *Let us define the logic function f by the following truth table.*

A	B	C	D	$f(A,B,C,D)$
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

1. *(1.75 points) Write the product of sums expression of f.*

> ***Solution:*** *We write product of the sums where f is in state of* 0 *;* $f(A,B,C,D) = (A+B+C+\overline{D}).(A+\overline{B}+\overline{C}+D).(A+\overline{B}+\overline{C}+\overline{D}).(\overline{A}+B+C+\overline{D}).(\overline{A}+\overline{B}+C+\overline{D}).(\overline{A}+\overline{B}+\overline{C}+D).(\overline{A}+\overline{B}+\overline{C}+\overline{D})$**(0.25 x7)=1.75**

2. *($1\frac{1}{2}$ points) Give the maxterms and minterms expressions of f. Deduce the sum of products expression of f.*

> ***Solution:***
>
> $f(A,B,C,D) \overset{\textbf{(0.5)}}{\widehat{=}} \prod_M(1,6,7,9,D,E,F)$

$\overbrace{=}^{(\mathbf{0.5})} \sum_m(0, 2, 3, 4, 5, 8, A, B, C)$. *Then the sum of product expression is.*

$$f(A,B,C,D) = \overline{A}.\overline{B}.\overline{C}.\overline{D} + \overline{A}.\overline{B}.C.\overline{D}+$$
$$\overline{A}.\overline{B}.C.\overline{D} + \overline{A}.B.\overline{C}.\overline{D}+$$
$$\overline{A}.B.\overline{C}.D + A.\overline{B}.\overline{C}.\overline{D} + A.\overline{B}.C.\overline{D}+$$
$$A.\overline{B}.C.D + A.B.\overline{C}.\overline{D}.(\mathbf{0.5})$$

3. *(1.75 points) Using the Karnaugh map, simplify the sum of products expression of f.*

Solution:

CD \ AB	00	01	11	10
00	1	1	1	1
01	0	1	0	0
11	1	0	0	1
10	1	0	0	1

$$(\mathbf{0.25})\ f(A,B,C,D) = \underbrace{\overline{C}.\overline{D}}_{(\mathbf{0.5})} + \underbrace{\overline{A}.B.\overline{C}}_{(\mathbf{0.5})} + \underbrace{\overline{B}.C}_{(\mathbf{0.5})}.$$