

Exercise 01: Solve in \mathbb{R} the following equations and inequations:

(1) $x^2 - 3x + 1 = 0$	(2) $x^2 - 2\sqrt{3}x + 3 = 0$	(3) $2x^2 - 4x + 6 = 0$
(4) $x^2 - 3x + 1 > 0$	(5) $x^2 - 3x + 1 \leq 0$	(6) $x^2 - 3x + 6 > 0$
(7) $x^2 - 2\sqrt{3}x + 3 > 0$	(8) $\sqrt{x^2 - 3x} > 2$	(9) $\sqrt{x^2 + x - 2} > 1$

Exercise 02: Let P, Q, R be three assertions.

- (1) Draw up the truth table of the following assertion:

$$(A) : [(P \wedge Q) \Leftrightarrow \bar{R}] \Rightarrow [\bar{Q} \vee (Q \Rightarrow P)].$$

- (2) Without using the truth table let us show that this proposition is true.

$$(Q \wedge \bar{R}) \Rightarrow [(P \Rightarrow R) \wedge (\bar{P} \vee R)].$$

- (3) Say if the following assertions are true or false and write their negation.

(a) $\forall x \in \mathbb{R}, x < 5 \Rightarrow x^2 < 25.$

(b) $\forall x, y \in \mathbb{R}, x > y \Rightarrow x^2 > y^2.$

(c) $\forall x, y \in \mathbb{R}^+, x < y \Rightarrow \sqrt{x} < \sqrt{y}.$

(d) $(\forall x \in \mathbb{R}) (\exists y \in \mathbb{Z}); (3x + y \leq 0).$

(e) $\forall x, y \in \mathbb{R}, \exists n \in \mathbb{N}, nx > y.$

(f) $(\exists x \in \mathbb{R}) (\forall y \in \mathbb{N}); -5x + 2y > 1.$

(g) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}^*, x^2 + 2xy + 3 > 0.$

Exercise 03 : Prove that:

(1) $\forall n \in \mathbb{N}^*, \sqrt{n^2 + 1}$ is not an integer.

(2) (Additional) $\forall n \in \mathbb{N}^*, \sqrt{n+1} + \sqrt{n}$ is not an integer.

(3) $\forall n \in \mathbb{N}^*, 7^n + 6n - 1$ est un multiple de 12.

(4) $\forall n \in \mathbb{N}^*, 2^{n-1} \leq n!$, avec $n! = n(n-1)(n-2) \times \dots \times 1$ et $0! = 1.$

(5)

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}.$$

Exercise 04 :

(1) Let a and p be two natural integers, prove that :

$$(p \text{ prime integer and } p \text{ divide } a^2) \Rightarrow p \text{ divide } a.$$

(2) (a) If p is a prime integer then \sqrt{p} is an irrational number.

(b) deduce that, $\sqrt{2} + \sqrt{3}$ is irrational number.

(3) Prove that:

$$\sqrt{2} + \sqrt{3} + \sqrt{6} \notin \mathbb{Q}.$$

Sincere wishes you success (MESSIRDI BACHIR)