Abou Bakr Belkaïd University-University of Tlemcen
Faculty of Science
Department of Informatic

Worksheet N°2
The complex numbers
LMD 1st year 2023-2024

Exercise 1

1. Write in the "algebraic" form $(a+ib)$ the following complex numbers

$$\frac{3+6i}{3-4i}, \qquad (-\frac{1}{2}+\frac{\sqrt{3}}{2}i)^6, \qquad \frac{-2}{1-i\sqrt{3}}, \qquad \frac{1}{(1+2i)(3-i)} \qquad (\textbf{Optional}).$$

2. Write in the polar $(r(\cos\theta + i\sin\theta))$ and the exponential polar form $(re^{i\theta})$, the following complex numbers and there conjugate

$$-2, \qquad 3+3i, \qquad -1-i\sqrt{3}, \qquad \frac{1+\sqrt{3}i}{\sqrt{3}-i} \qquad (\textbf{Optional}).$$

3. Prove that

$$(\cos(\frac{\pi}{7})+i\sin(\frac{\pi}{7}))(\frac{1-i\sqrt{3}}{2})(1+i) = \sqrt{2}(\cos(\frac{5\pi}{84})+i\sin(\frac{5\pi}{84}))$$

$$(1-i)(\cos(\frac{\pi}{5})+i\sin(\frac{\pi}{5}))(\sqrt{3}-i) = 2\sqrt{2}(\cos(\frac{13\pi}{60})-i\sin(\frac{13\pi}{60})) \qquad (\textbf{Optional})$$

Exercise 2

Let $a=\sqrt{3}+i$ and $b=\sqrt{3}-1+i(\sqrt{3}+1)$ two complex numbers,

1. Check that $b=(1+i)a$.
2. Deduce that $|b| = 2\sqrt{2}$ and $\arg(b) = \frac{5\pi}{12}$ $[2\pi]$.
3. Deduce from the above that: $\cos(\frac{5\pi}{12}) = \frac{\sqrt{6}-\sqrt{2}}{4}$.

Exercise 3

1. Find the squar roots for a complex number

$$-1, \qquad i, \qquad 3-4i, \qquad \frac{\sqrt{3}+i}{2} \qquad (\textbf{Optional})$$

2. Find $z \in \mathbb{C}$ such that

$$z^2+z+1=0, \qquad z^3+8=0, \qquad z^4+i=0, \qquad z^5=\overline{z} \qquad (\textbf{Optional})$$

Exercise 4

Let 'f' be a function defined from $\mathbb{C}$ to $\mathbb{C}$, by

$$\forall z \in \mathbb{C}, \qquad z \neq -i, \qquad f(z) = \frac{1-z}{1-iz}$$

1. Find $z \in \mathbb{C}$ such that $f(z) \in \mathbb{R}$
2. Find $z \in \mathbb{C}$ such that $f(z) \in i\mathbb{R}$.

Exercise 5

Determine in each case, the set of points $M(x, y)$, with affix $z = x + iy$ such that:

1. $|z - (2 - i)| = \sqrt{2}$.
2. $|z - 1 - 2i| = |z + 2 - i|$.
3. $|\overline{z} - 2i| = |z + 2|$. (**Optional**).